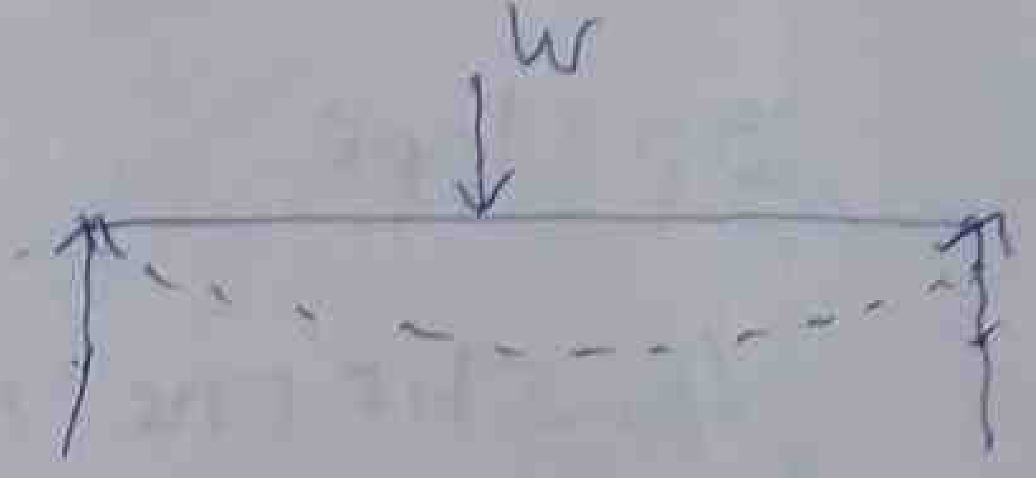
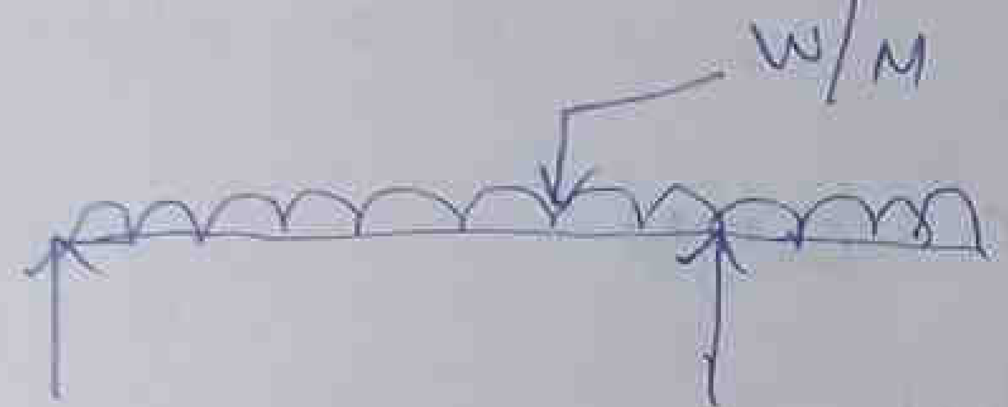
UNIT - I

Slope and Deflection

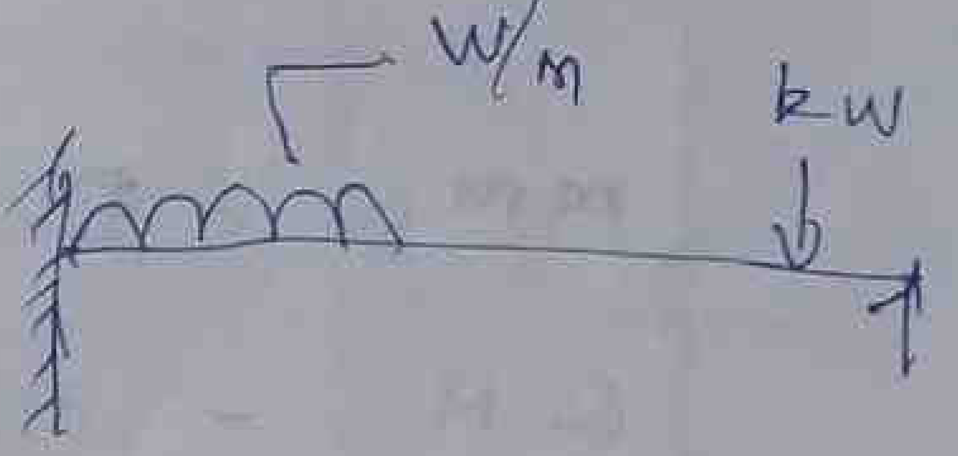
(a) Cantilever beam



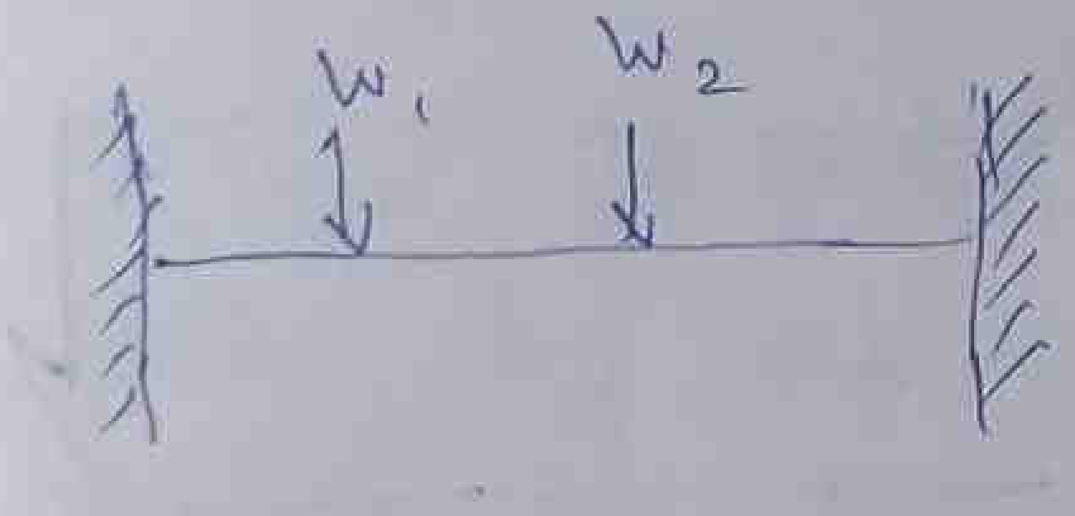
(b) Simply supported beam.



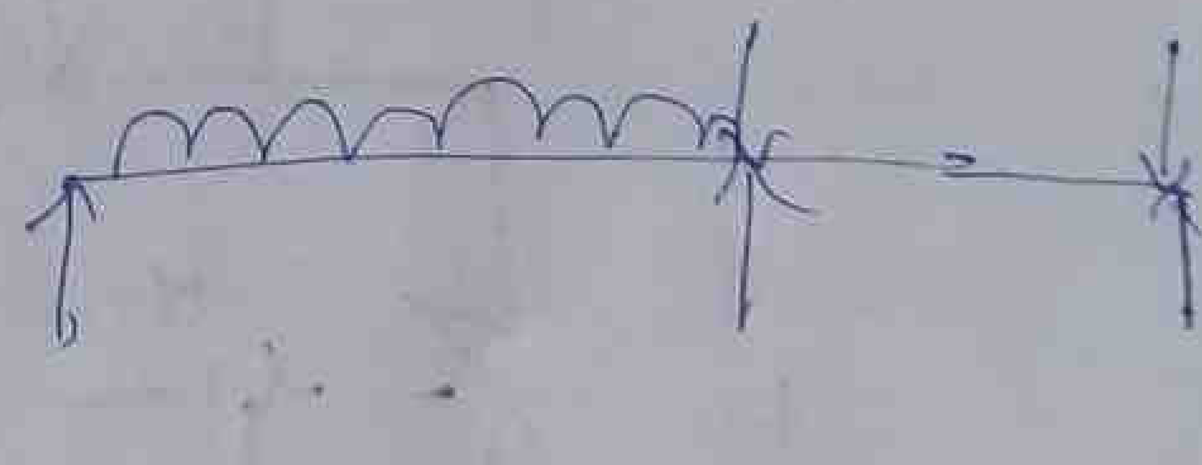
(c) overhanging beam



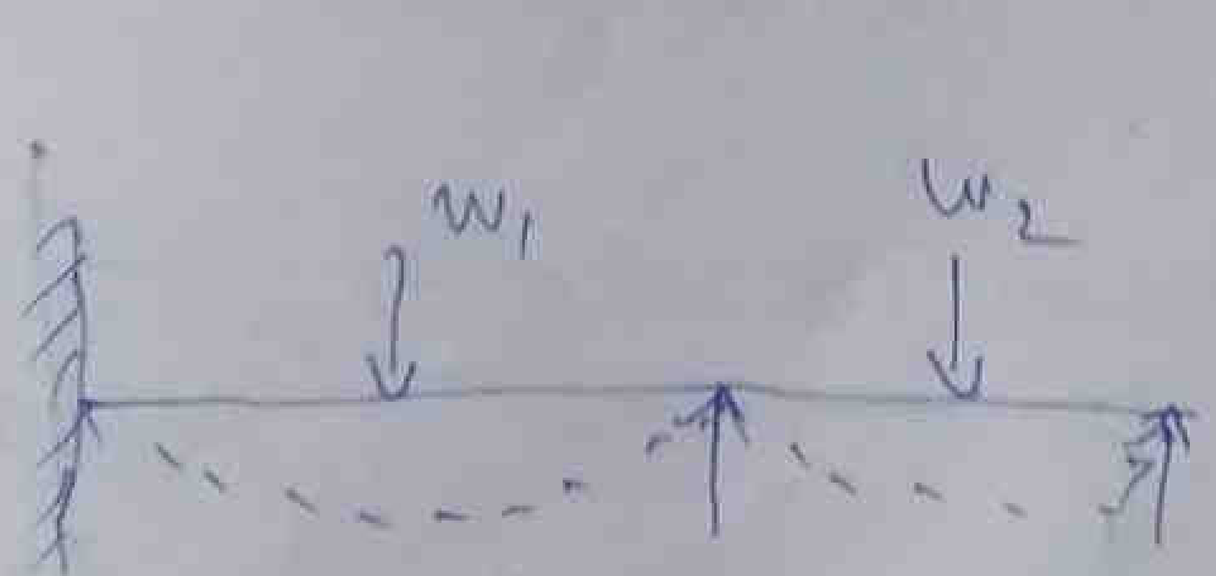
(d) propped beam.



(e) fixed beam



(F) Continuous beam.



(g) Continuous beam.

Deflection Shape and elastic Curves of beam

Terms

1. Elastic curves

2. Deflection

3. slope.

4. Stiffness.

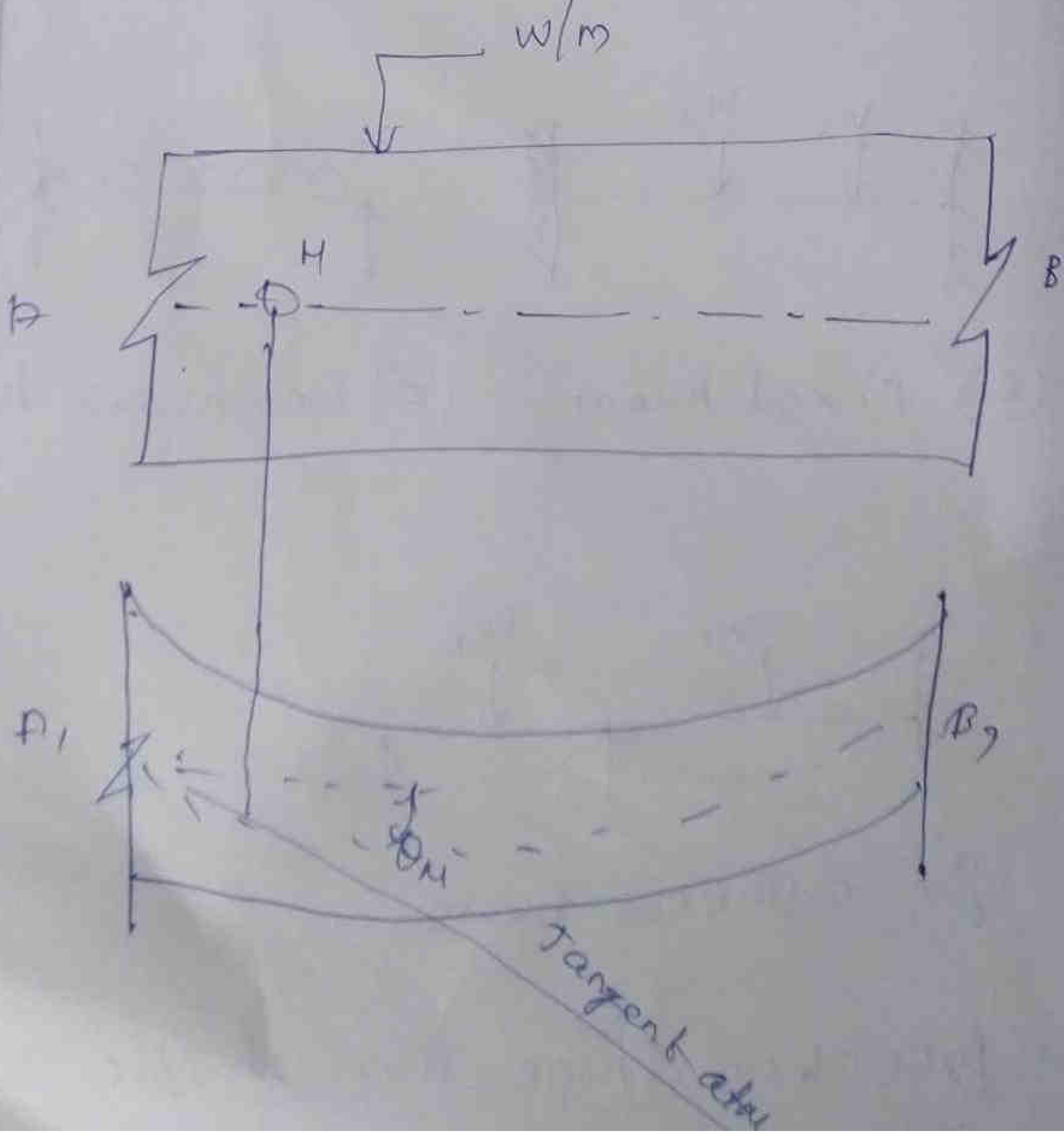
AMB - Neutral curve axis.

$A_1 M_1 B_1$ - Elastic curve.

MM_1 - Deflection at M

θ_M - Slope at M.

Diagram



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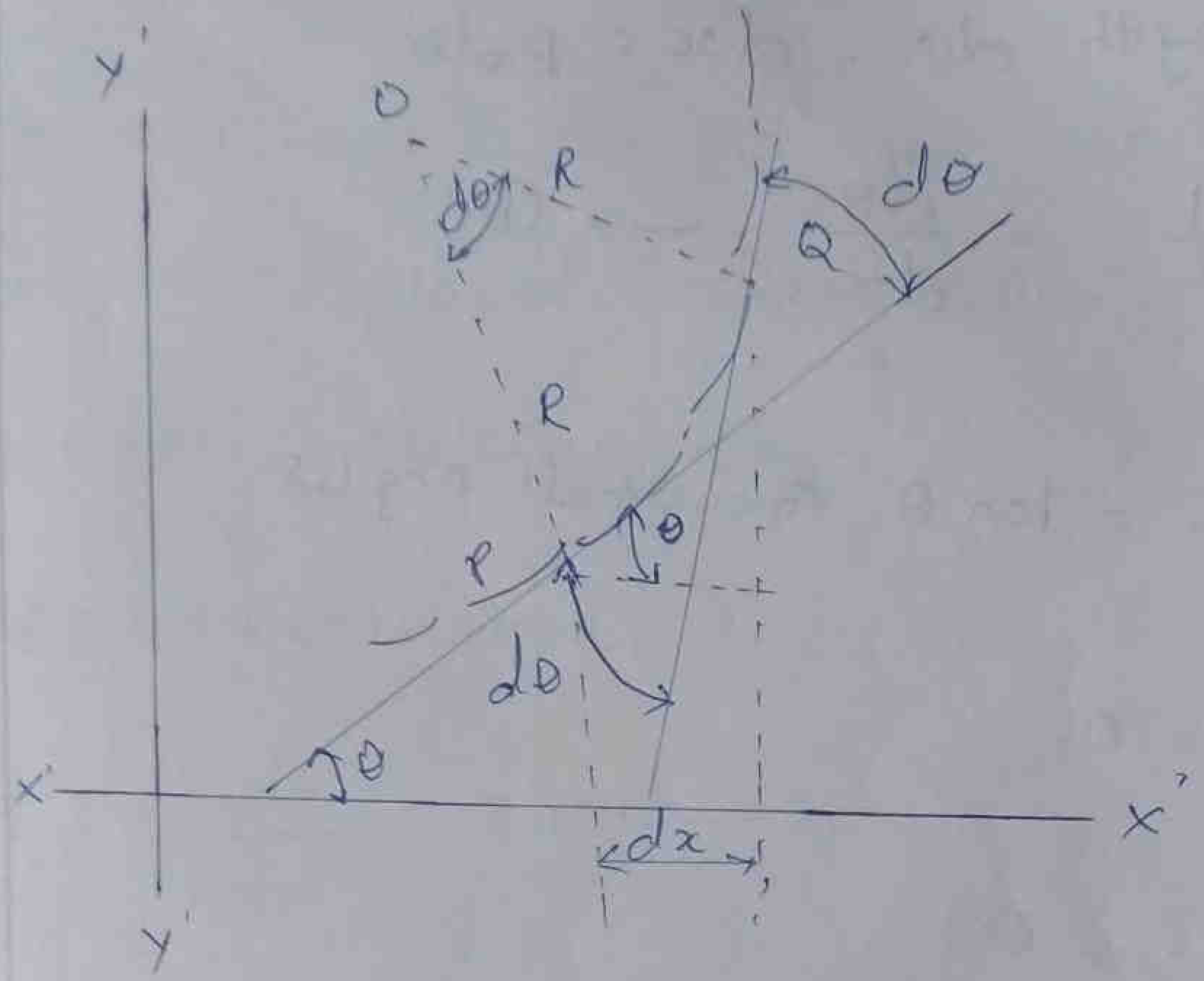
Euler - Bernoulic general differential equation.



PROCEDURE:

* O be the centre of the elastic curve.

* R be the Radius of the elastic curve.

* θ and $(\theta + d\theta)$ be the angles made by the tangents at P & Q with x-axis.

* ds - length of Arc PQ.

* $d\theta$ - Angle bounded by arc ds at centre O.

* P & Q are very close.

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Equation:-

Arc length $ds = dx = R\,d\theta$

$$\frac{1}{R} = \frac{d\theta}{dx} \longrightarrow (1)$$

$\frac{dy}{dx} = \tan\theta$ for small angles.

$\tan = \theta$.

$$\frac{dy}{dx} = \theta.$$

Differentiate with respect 'x'

$$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} \longrightarrow (2)$$

$$\frac{1}{R} = \frac{d^2y}{dx^2} \longrightarrow (3)$$

$$\frac{1}{R} = \frac{M}{EI} \longrightarrow (4)$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (\text{or}) \quad EI\frac{d^2y}{dx^2} = M.$$

This is the Euler Bernoulli Flexural differential equation.

⇒ ⑥ slope $\left(\frac{dy}{dx}\right)$ and deflection (y) of beams.

⑤ give shear force is rate of loading they can placed in the following order to find deflection, slope.

EIy = Deflection (δ)

$EI \frac{dy}{dx}$ = slope

$EI \frac{d^2y}{dx^2}$ = Bending moment (M)

$EI \frac{d^3y}{dx^3} = \frac{dM}{dx}$ = shear force (F)

$EI \frac{d^4y}{dx^4} = \frac{d^2M}{dx^2} = \frac{dF}{dx}$ = Rate of loading (w).

Method of Euler Bernoulic differential equation

1) Double integration method.
2) Macaulay's method.
3) Mohr's area moment methods.
4) strain energy (or) unit load method.
5) conjugated beam method.

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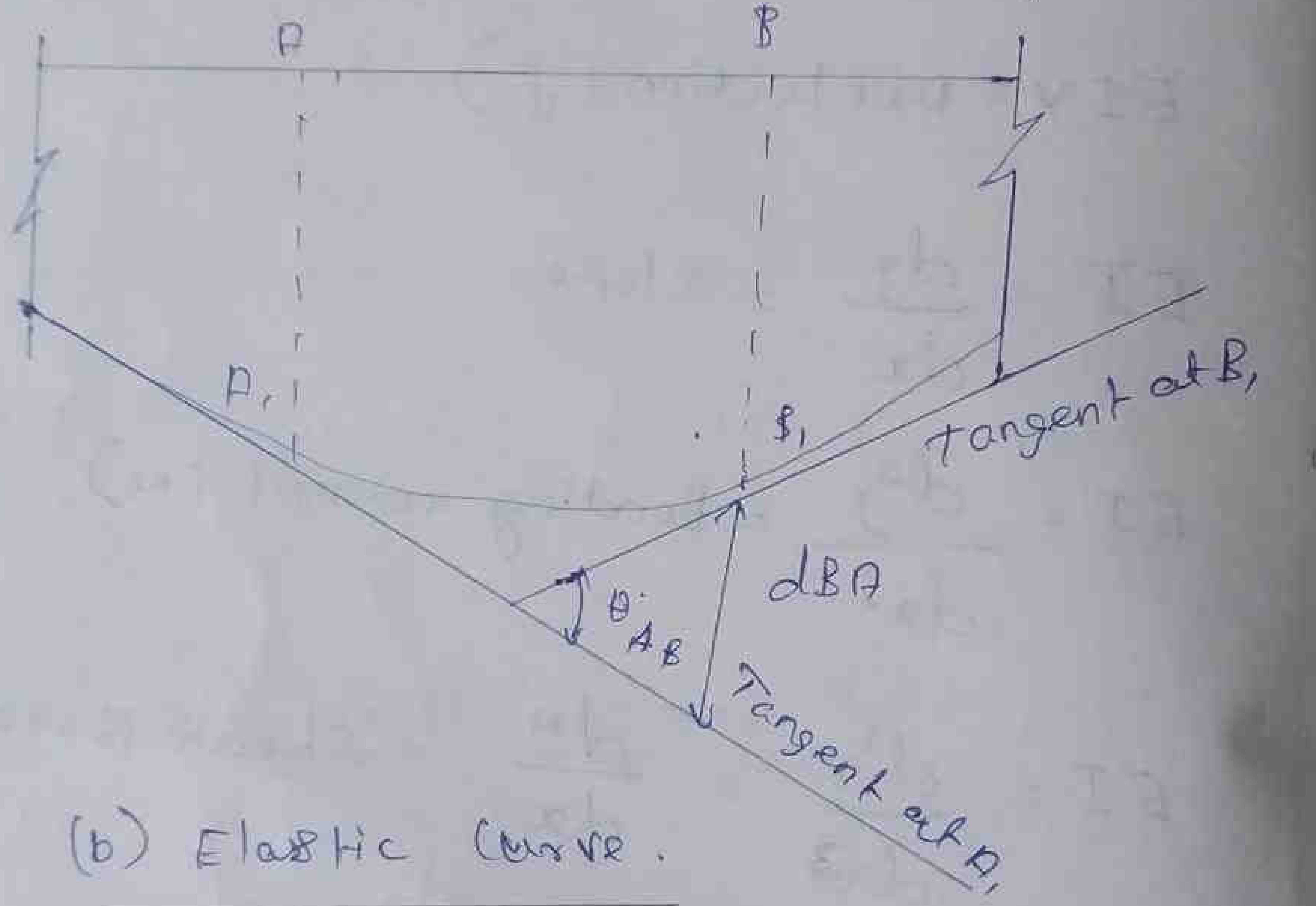
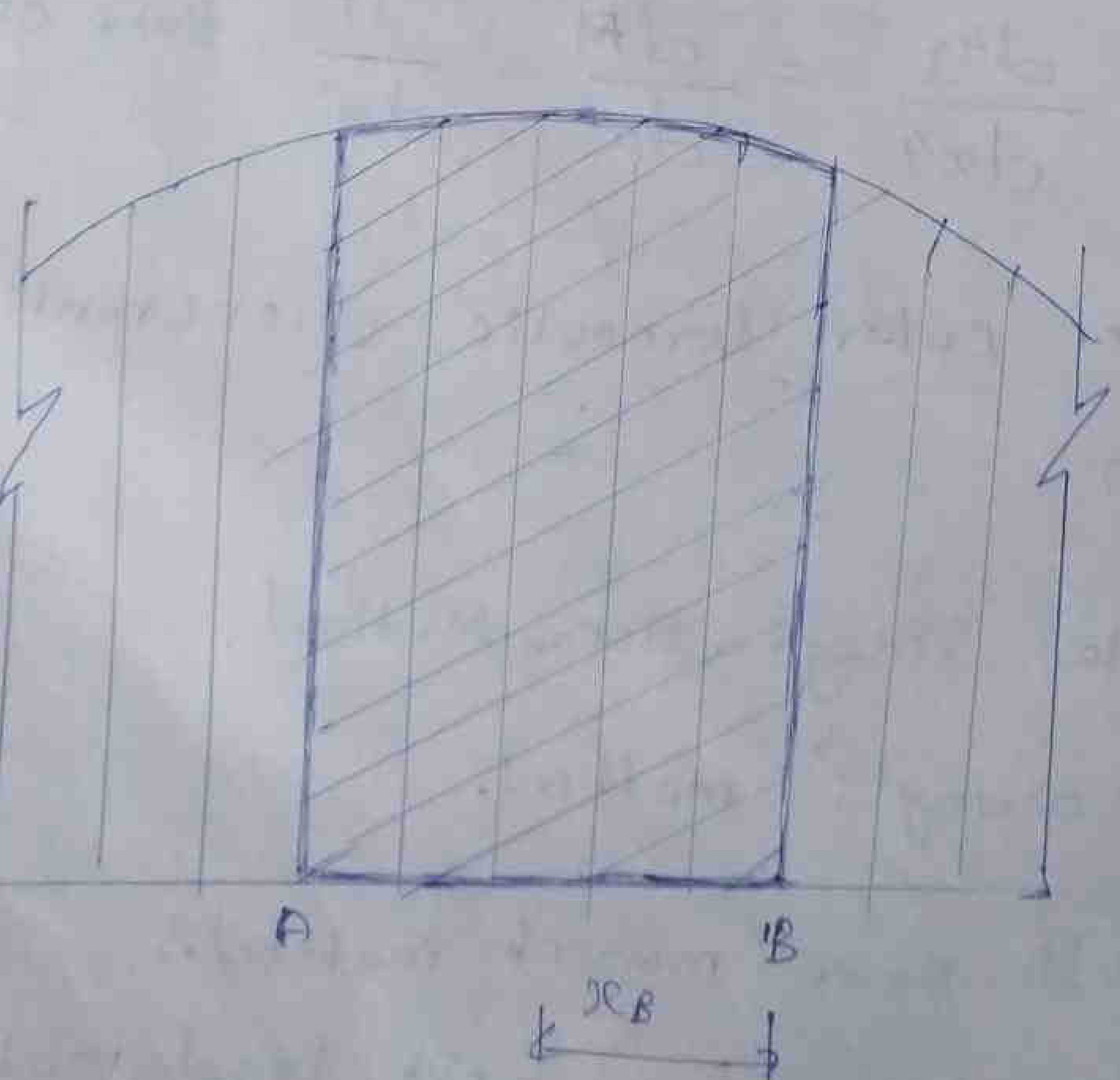
Mohr's Area moment theorems for slope and deflection

(a) Loaded Beam.



(b) Elastic Curve.



B.M diagram.

Mohr's area moment theorems

Theorem : 1

The change in angle of slope b/w the tangents at any two points (A & B) on the elastic curve is equal to the area OF BMD in between two points divided by Flexural rigidity (EI).

$$\boxed{\theta_{AB} = \frac{a_{AB}}{EI}}$$

where, θ_{AB} is in radius.

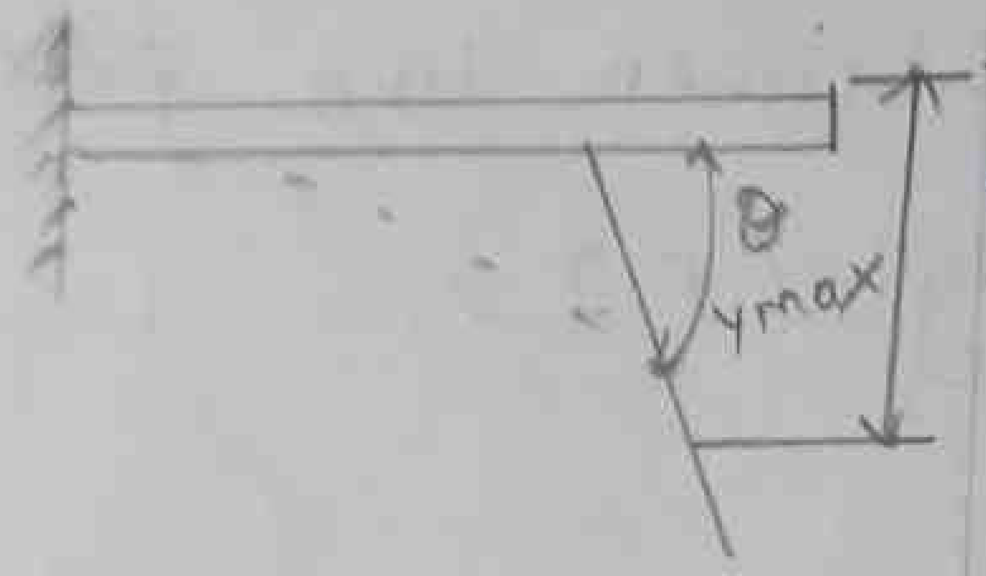
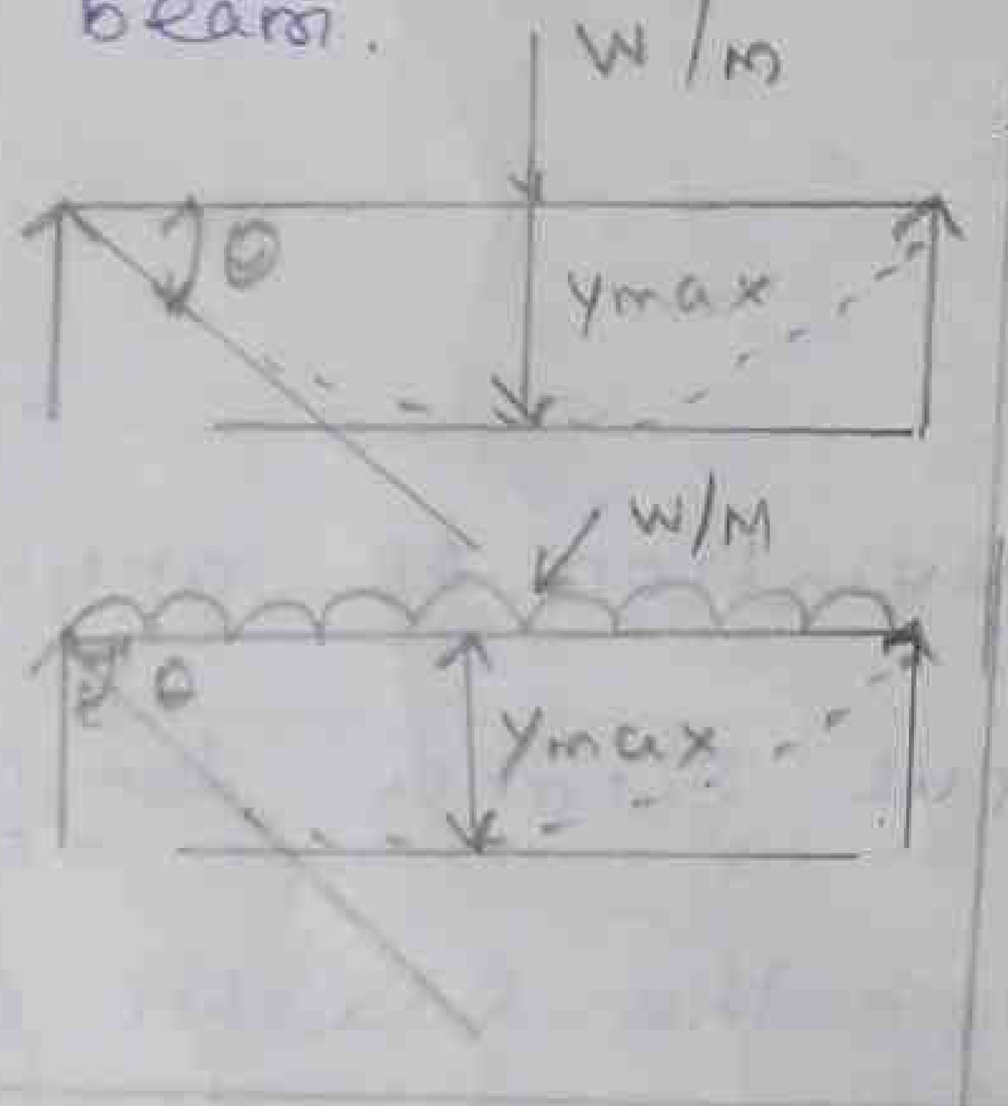
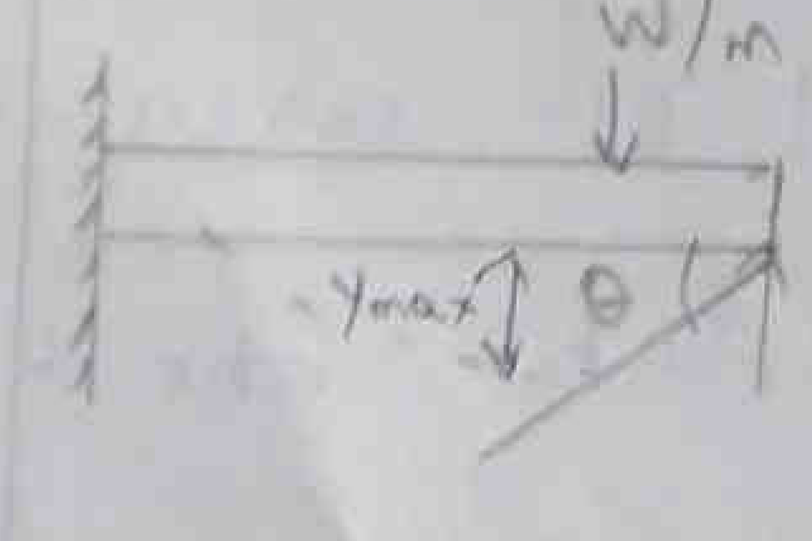
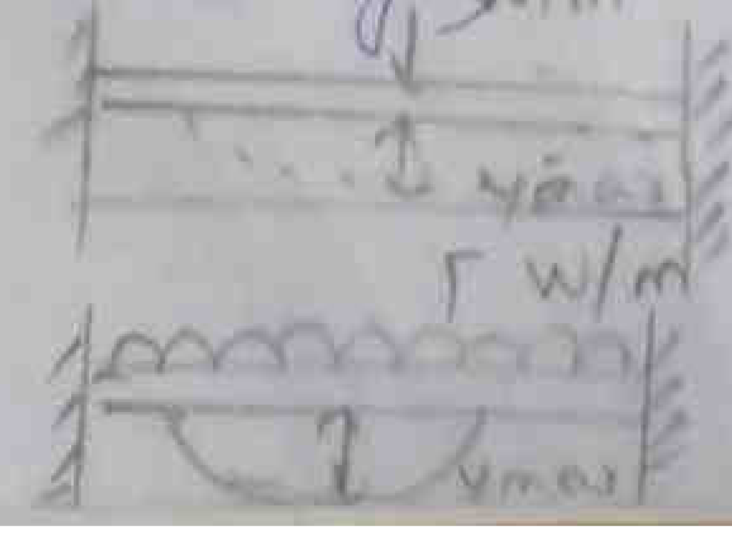
Theorem : 2

The Tangential deviation OF any Point (B) on the elastic curve From a tangent at any other point (A) on the elastic curve ⊥ to the original axis of the beam is equal to the moment OF area OF BMD in b/w those two points about B divided by Flexural rigidity (EI)

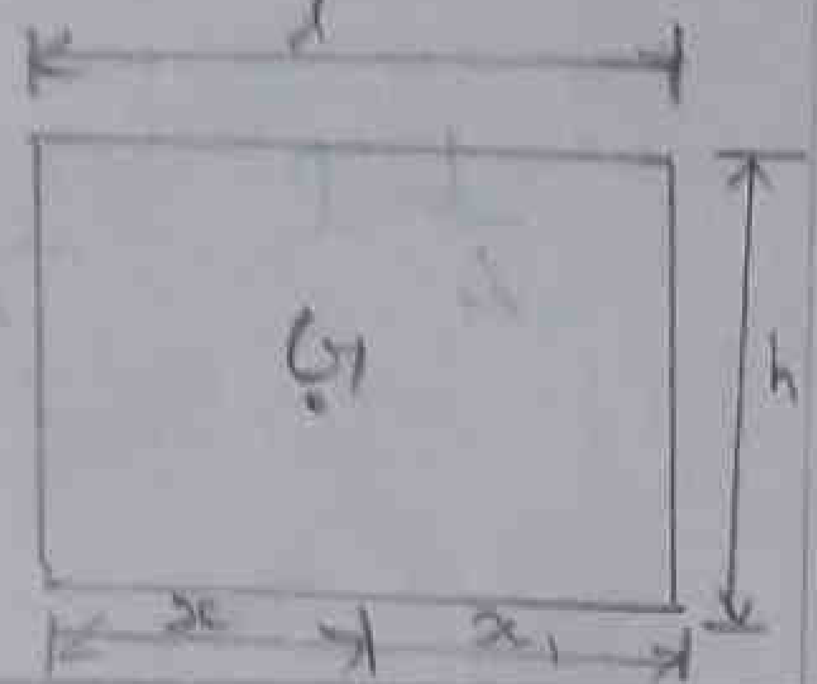
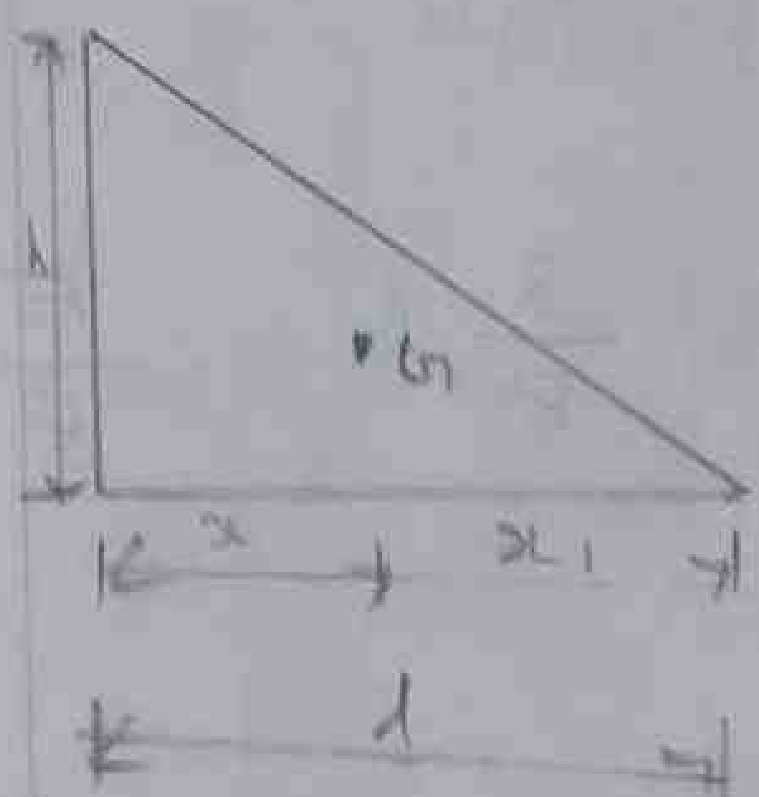
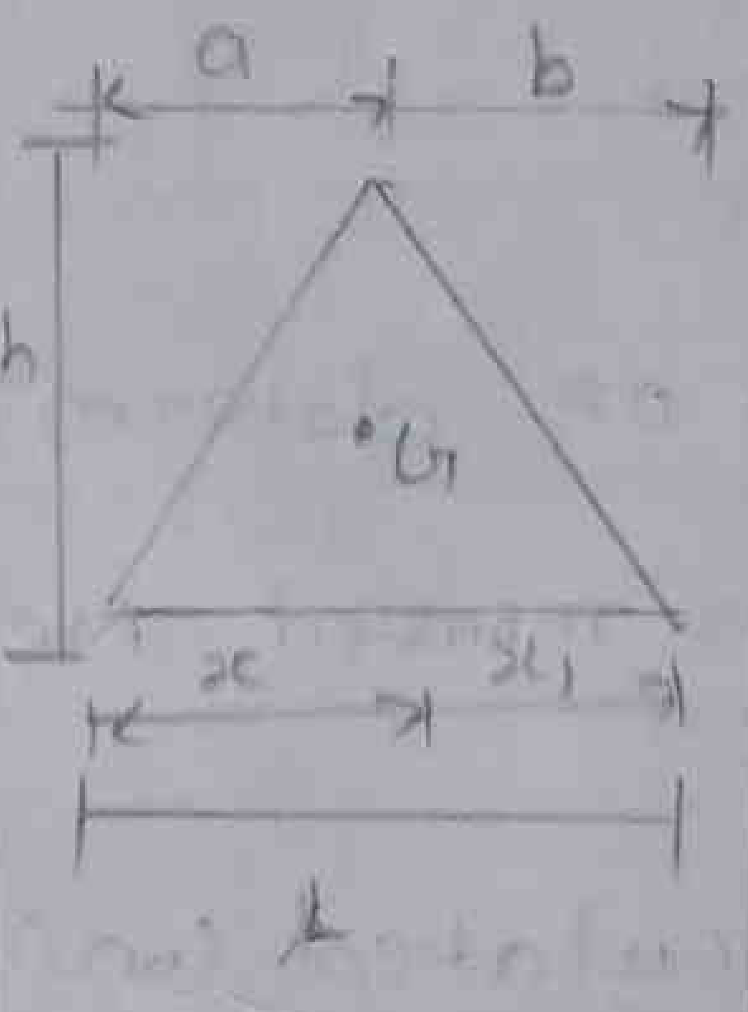
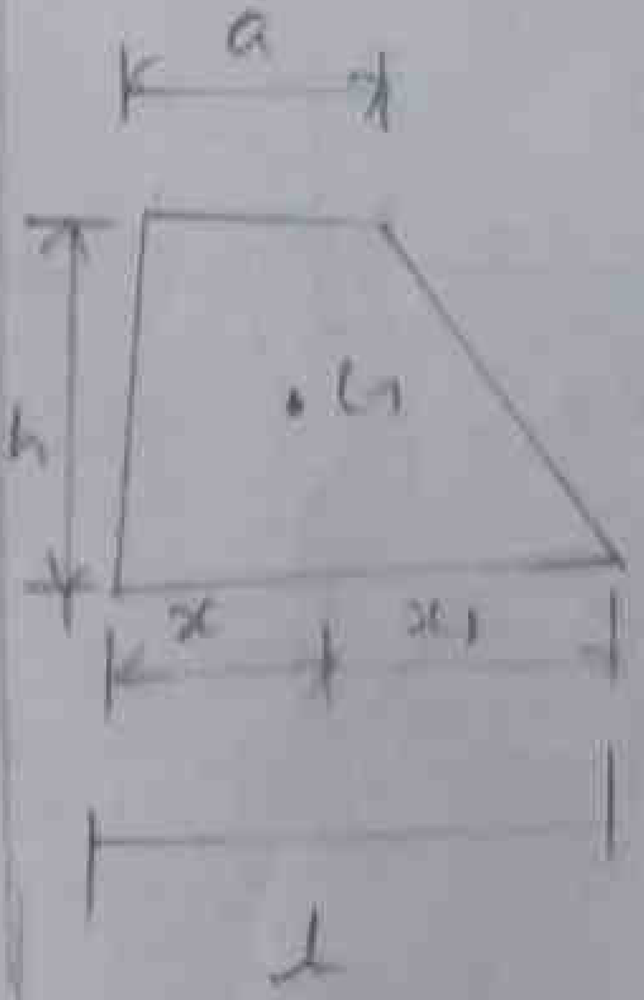
$$\boxed{d_{BA} = \frac{a_{AB}\, x_B}{EI}}$$

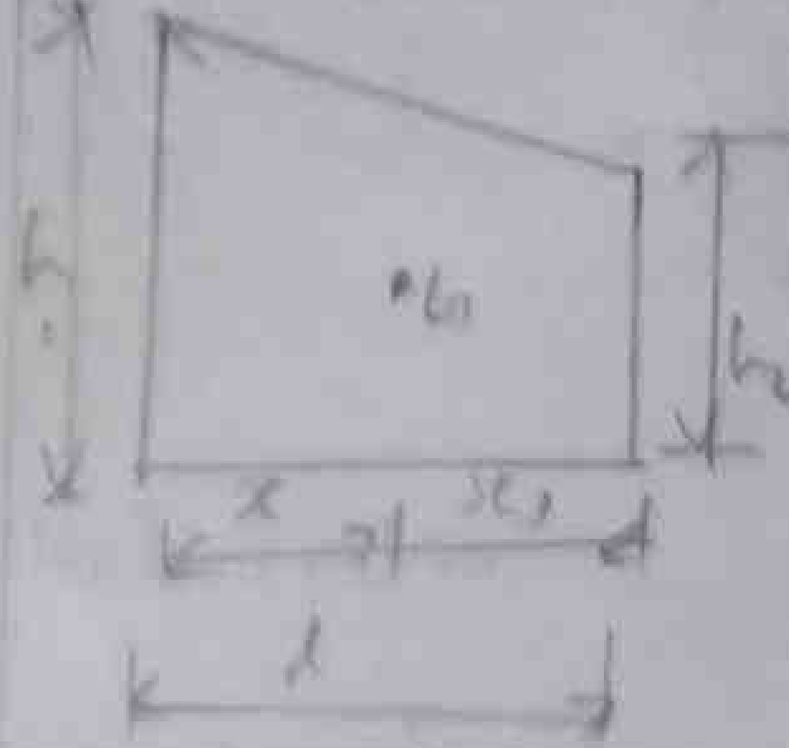
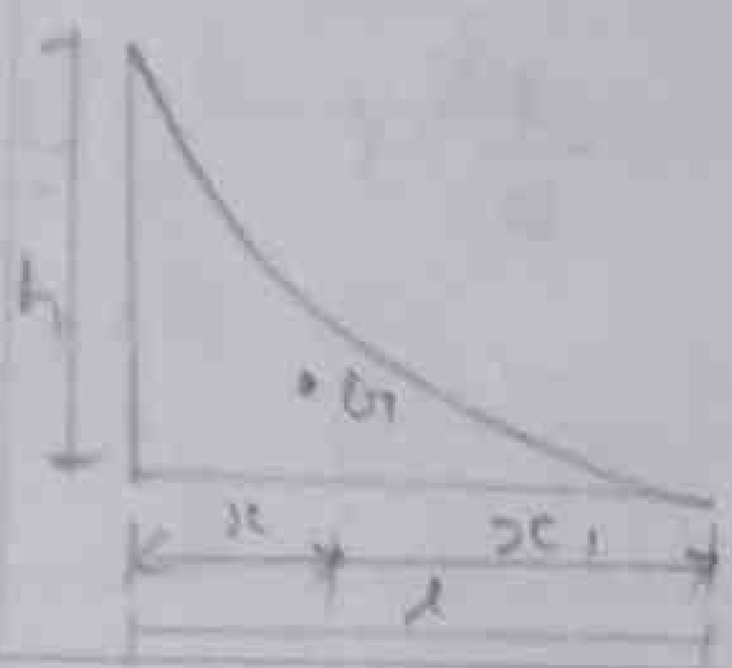
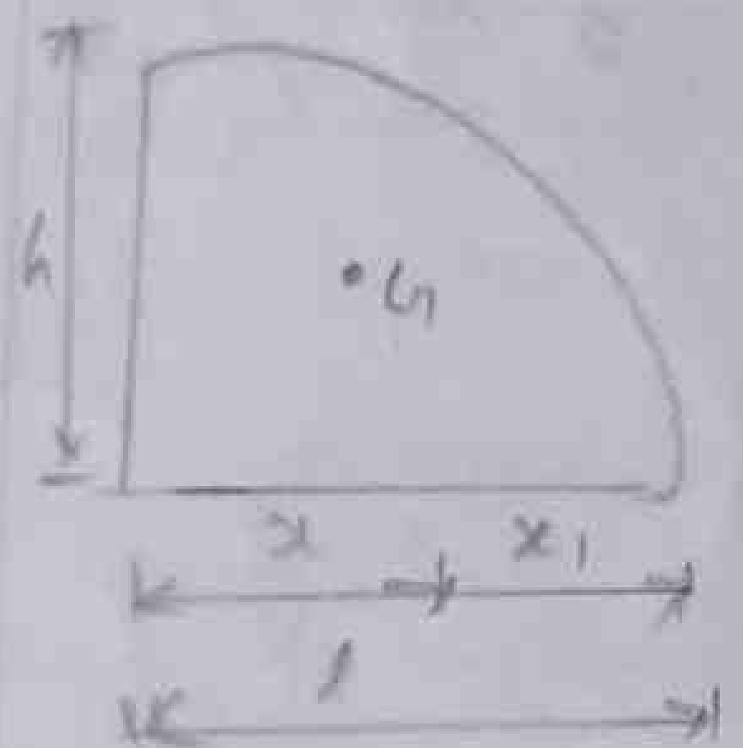
wednes day

1. Elastic curve & zero slope points in beam.

S.NO	Type of beam	Max, & min points of	
		Slope	Deflection
1.	Cantilever beam.	zero @ Fixed end	Zero @ Fixed end.
		Max @ ~~Fixed~~ Free end	max @ Free end
2.	Simply Supported beam.	zero @ mid span	Zero @ Support
		max @ Support	max @ mid ~~Support~~ span
3.	propped Cantilever	Zero @ Fixed end	zero @ Support
		max @ propped end	max, under load point.
4	Fixed beams (symmetrically loading)	Zero @ Supports	zero @ supports.
		zero @ mid span.	max @ mid span.

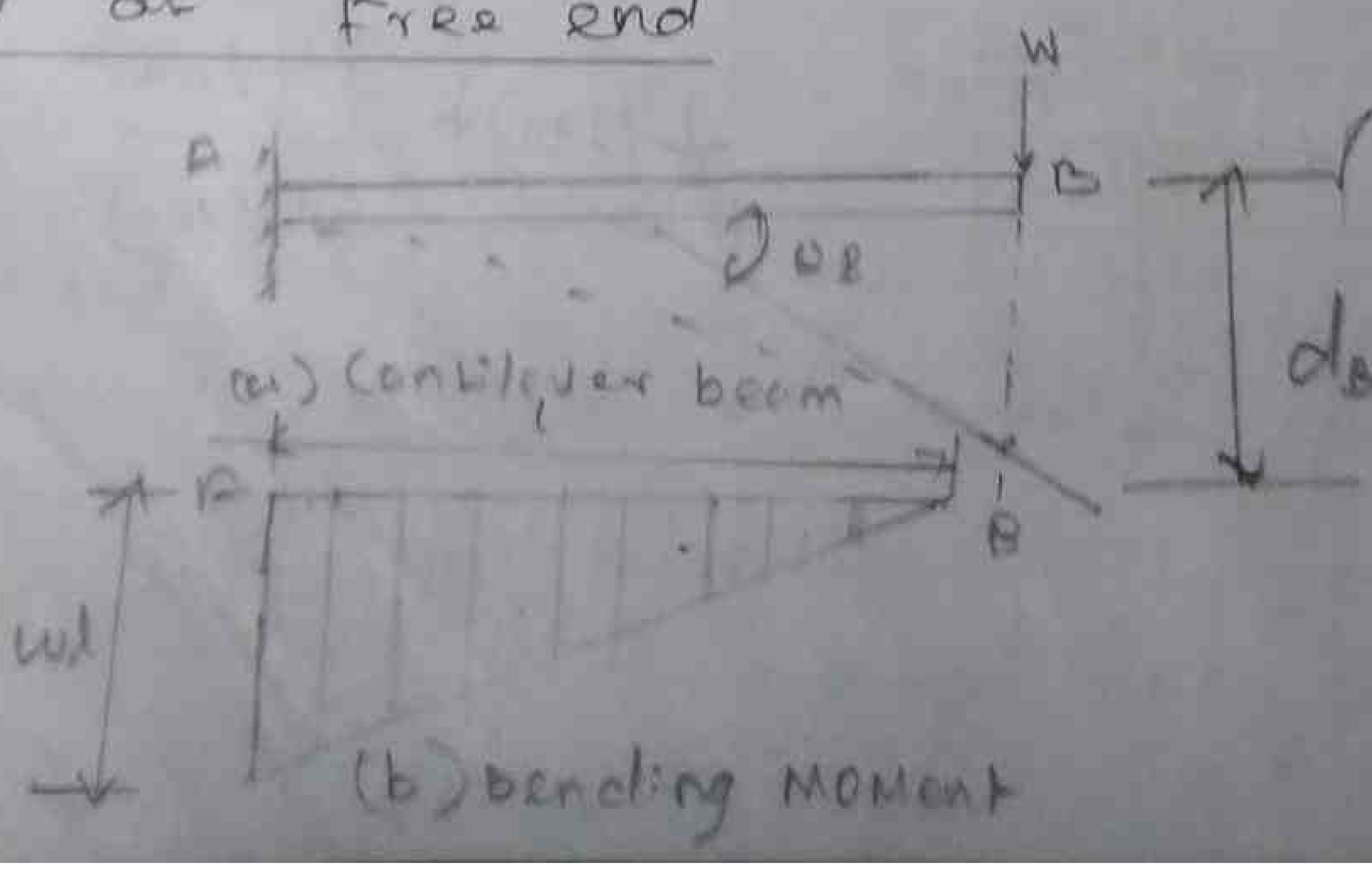
2) Bending moment Diagrams, their area and centroid distances:

S.NO	Shape	Area	Centroid distances	
		A	x	x_1
1		lh	$\frac{1}{2}l$	$\frac{1}{2}l$
2		$\frac{1}{2}lh$	$\frac{1}{3}l$	$\frac{2}{3}l$
3.		$\frac{1}{2}lh$	$\frac{1}{3}(l+a)$	$\frac{1}{3}(l+b)$
4.		$\frac{1}{2}(l+a)h$	$\frac{l^2+la+a^2}{3(l+a)}$	$\frac{2l^2+2la-a^2}{3(l+a)}$

	$\frac{1}{2}l(h_1+h_2)$	$\frac{1}{3}l\left[\frac{h_1+2h_2}{h_1+h_2}\right]$	$\frac{1}{3}l\left[\frac{2h_1+h_2}{h_1+h_2}\right]$
	$\frac{1}{3}lh$	$\frac{1}{4}l$	$\frac{3}{4}l$
	$\frac{2}{3}lh$	$\frac{3}{8}l$	$\frac{5}{8}l$

Slope and deflection of determinate beams by mohr's area moment Method.

1) Cantilever with calculated concentrated load at free end



- Theorems:

θ_{AB} = Shape at B. & Targent diviation

d_{BA} = deflection at B.

Area of BMD in between A & B $\Big\}$ $a_{AB} = \frac{1}{2} \times l \times wl$

$= \frac{wl^2}{2}$

Applying Mohr's area moment theorem

Slope at B = $\frac{\text{Area of BMD in b/w A \& B.}}{EI}$

$\theta_B = \frac{a_{AB}}{EI}$

$\theta_B = \frac{\frac{wl^2}{2}}{EI}$

$\theta_B = \frac{wl^2}{2} \times \frac{1}{EI}$

$$\boxed{\theta_B = \frac{wl^2}{2EI}}$$

Centroid of BMD from B.

$\Rightarrow x_B = \frac{2l}{3}$.

Deflection at $B = \frac{a_{AB} \quad x_B}{EI}$

(or)

$$y_B = \frac{wl^2}{\cancel{2}EI} \times \frac{\cancel{2}l}{3}$$

$$\boxed{y_B = \frac{wl^3}{3EI}}$$

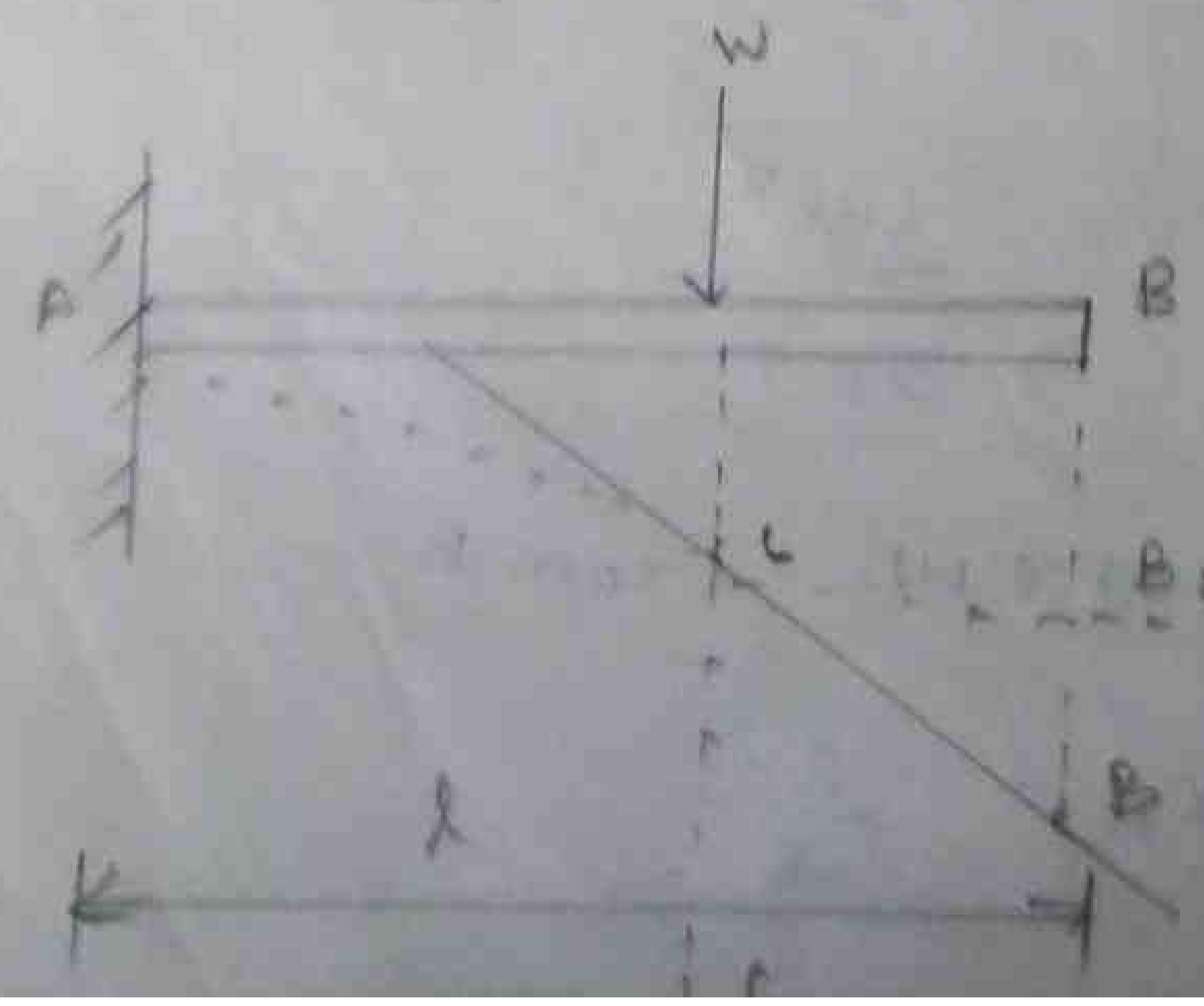
slope at $A\theta = 0$

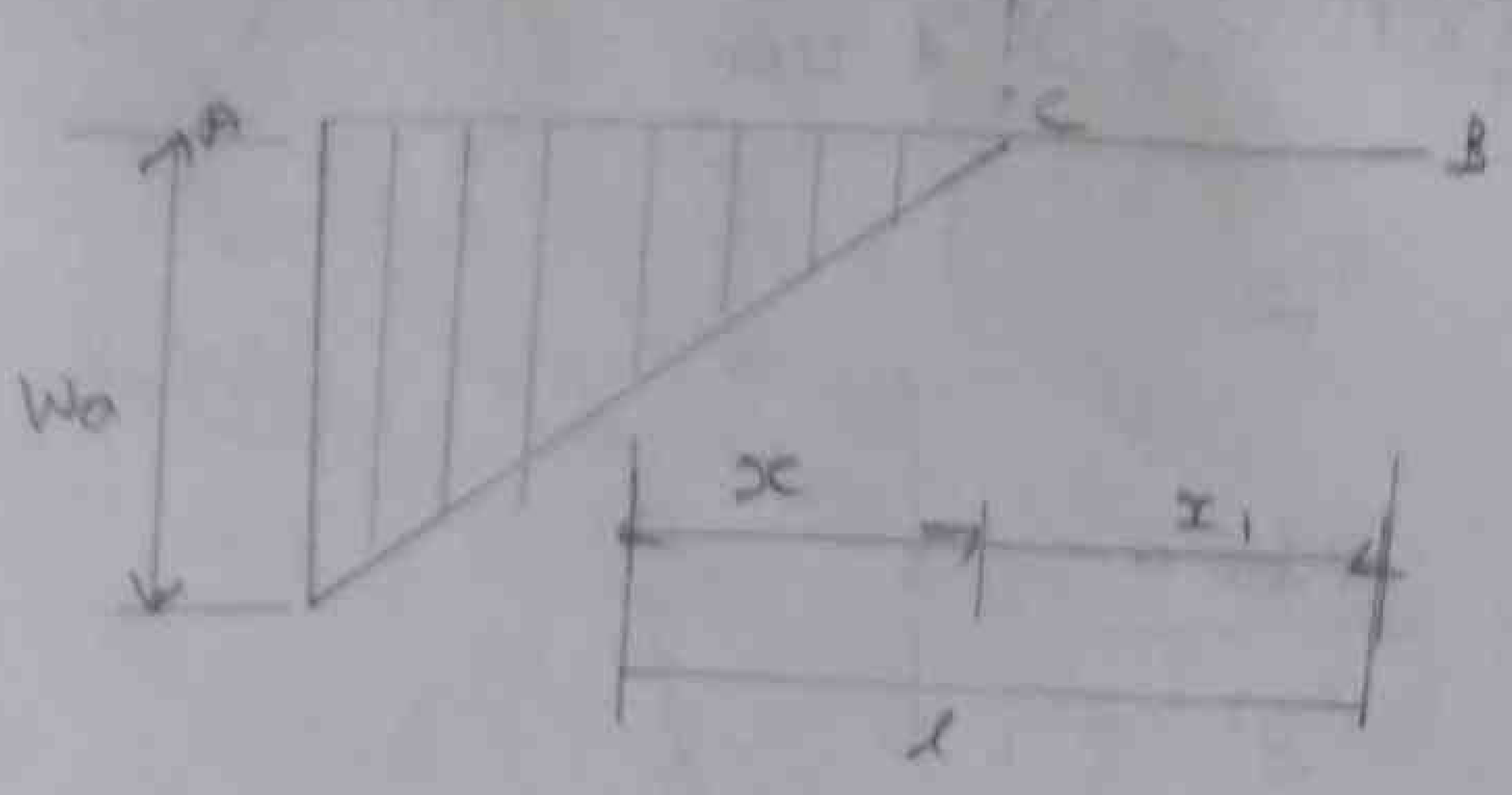
slope at $B\theta = \frac{wl^2}{2EI}$

deflection at $A\theta = 0$

deflection at $y_B = \frac{wl^3}{3EI}$

2) Contilever with concentrated load at a distance from 'a' from fixed end 'A'





slope at C = slope at B

Slope at A = 0.

In b/w A & C, Area of BMD.

$$a_{AC} = \frac{1}{2} \times a \times Wa.$$

$$= \frac{Wa^2}{2}$$

Its centroid from C: $x_c = \frac{2}{3} a.$

$$\text{slope at } C = \theta_c = \frac{a_{AC}}{EI} = \frac{Wa^2}{2EI} = \theta_B$$

$$\text{Deflection at } C = y_c = \frac{a_{AC}\, x_c}{EI} = \frac{Wa^2}{2EI} \times \frac{2}{3}a$$

$$= \frac{Wa^3}{3EI}$$

$$\text{Deflection at } B = a_{AB} = a_{AC} = \frac{Wa^2}{2}$$

$$x_B = (l-a) + \frac{2}{3}a$$

$$x_B = \frac{(3l - 3a) + 2a}{2}$$

$$x_B = \frac{3l - a}{3}.$$

$$y_B = \frac{a_{AB}\, x_B}{EI}$$

$$= \frac{wa^2}{2EI} \times \frac{3l - a}{3}.$$

$$= \frac{wa^2 (3l - a)}{6EI}$$

= <u>Slope</u>

$\theta_A = 0$

$\theta_B = \frac{wa^2}{2EI}$

$\theta_C = \frac{wa^2}{2EI}$

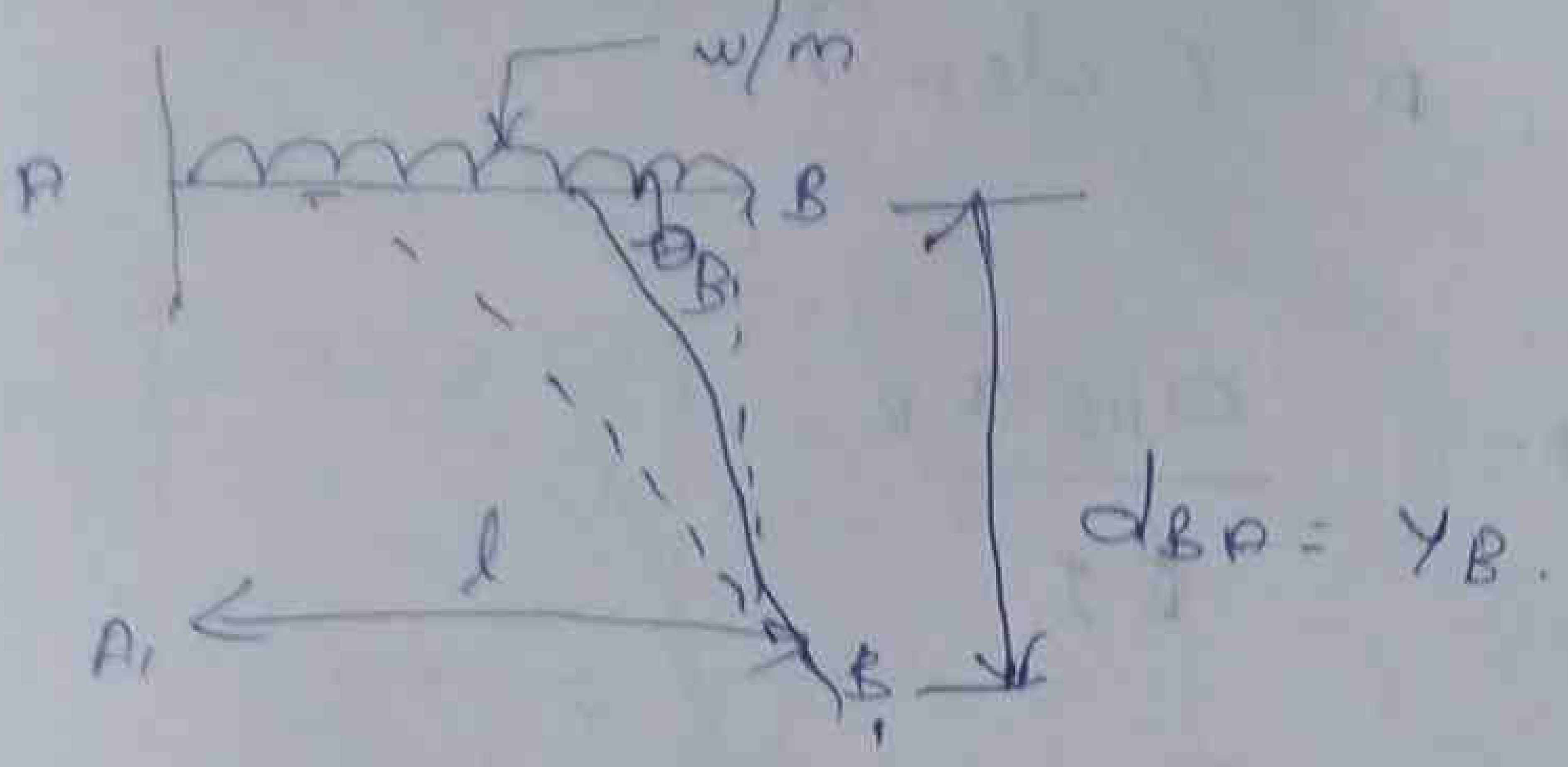
<u>Deflection</u>

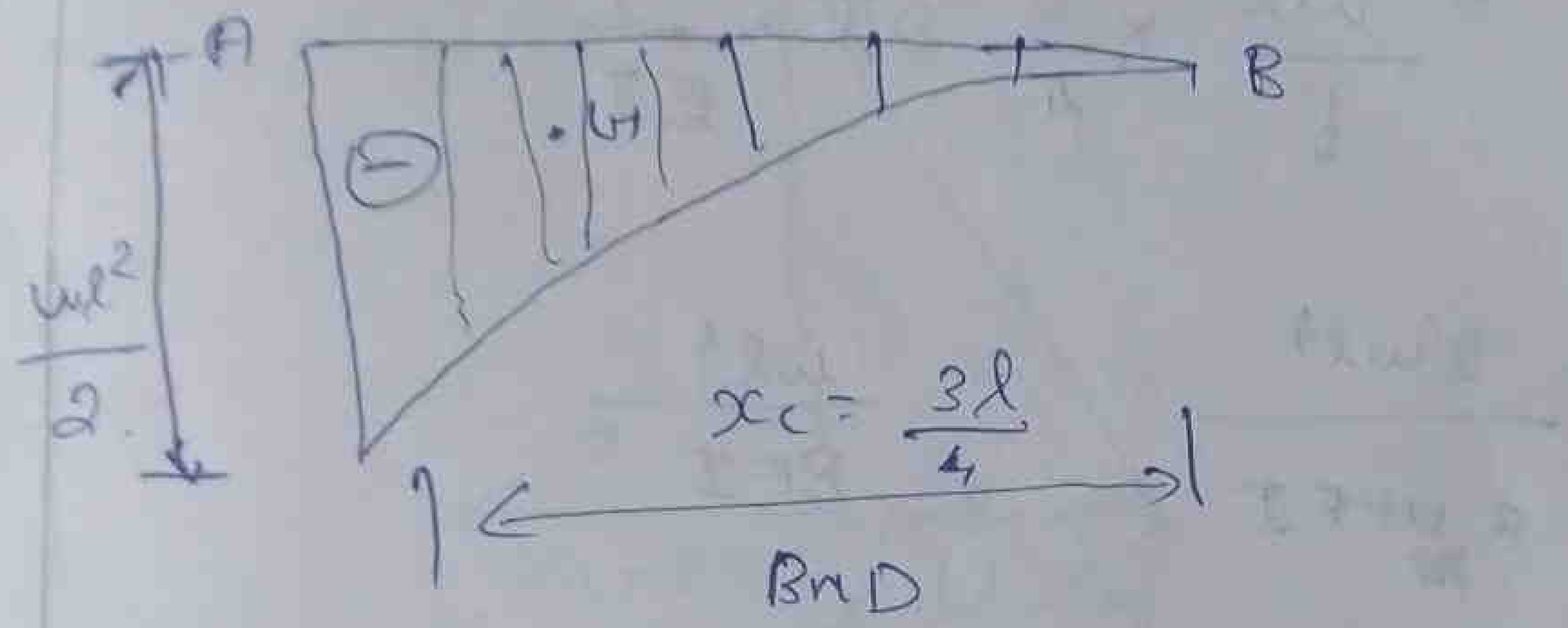
$y_A = 0$

$y_B = \frac{wa^2}{6EI}(3l - a)$

$y_C = \frac{wa^3}{3EI}$

18.3.22 Cantilever with udl throughout





Calculation :

Area of BMD b/w A & B.

$$a_{AB} = \frac{1}{3} \times l \times \frac{wl^2}{2}$$

$$= \frac{wl^3}{6}$$

Centroid from B, $x_B = \frac{3}{4} l$

Slope at A, $\theta_A = 0$.

Slope at B, $\theta_B = \frac{a_{AB}}{EI} = \frac{wl^3}{6EI}$

Deflection at B = Tangentical deviation

B w.r.t. A (d_{BA})

$$d_{BA} = y_B = \frac{a_{AB}\, x_B}{EI}$$

$$= \frac{wl^3}{6} \times \frac{3}{4} l \times \frac{1}{EI}$$

$$= \frac{\cancel{3} wl^4}{8 \cancel{24} EI} = \frac{wl^4}{8EI}$$

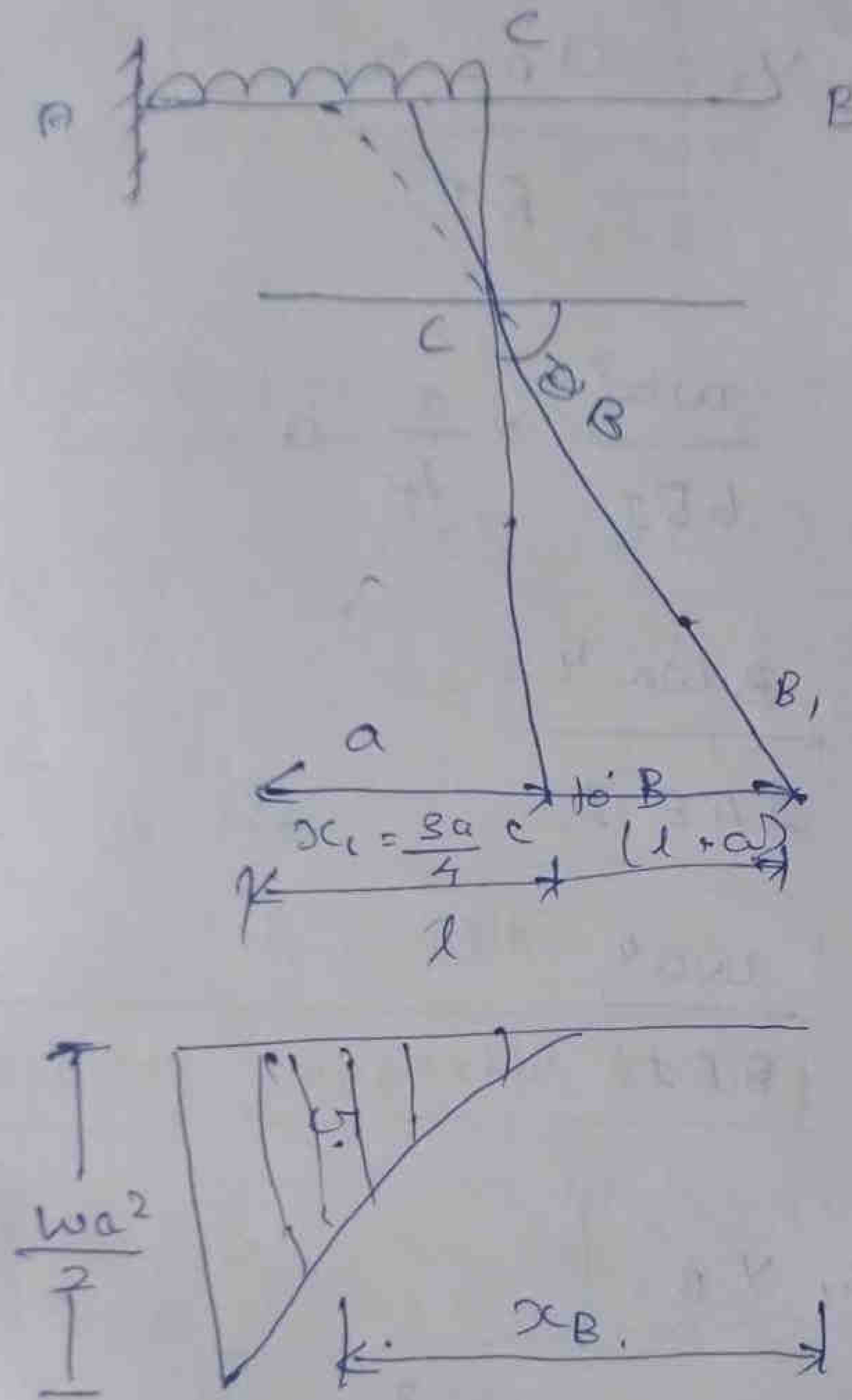
Slope at A, $\theta_A = 0$

Slope at B, $\theta_B = \frac{wl^3}{6EI}$

deflection at A, $y_A = 0$

deflection at B, $y_B = \frac{wl^4}{8EI}$

Cantilever with part udl For a distance 'a' From Fixed end A.



BM is zero in b/w C & B.

slope at C = Slope at A & Slope at B = 0

Area OF BMD

$$a_{AC} = \frac{1}{3} a \times \frac{wa^2}{2}$$

$$= \frac{wa^3}{6}$$

Centroid C, $x_c = \frac{3}{4} a$.

Slope c, $\theta_c = \frac{a_{AC}}{EI} = \frac{wa^3}{6EI} = \theta_B$.

Deflection at c, $y_c = \frac{a_{AC}\,\bar{x}_c}{EI}$

$$= \frac{wa^3}{6EI} \times \frac{3}{4} a.$$

$$= \frac{3wa^4}{24EI}.$$

$$= \frac{wa^4}{8EI}$$

Deflection at B, y_B

$$a_{AB} = a_{AC} = \frac{wa^3}{6}$$

$$\bar{x}_B = (l-a) + \frac{3}{4} a.$$

$$= \frac{4l - 4a + 3a}{4}.$$

$$= \frac{4l - a}{4}$$

$$y_B = \frac{a_{AB}\,\bar{x}_B}{EI} = \frac{wa^3}{6} \times \frac{4l-a}{4} \times \frac{1}{EI}$$

$$Y_B = \frac{Wa^3 (4l - a)}{24EI}$$

slope.

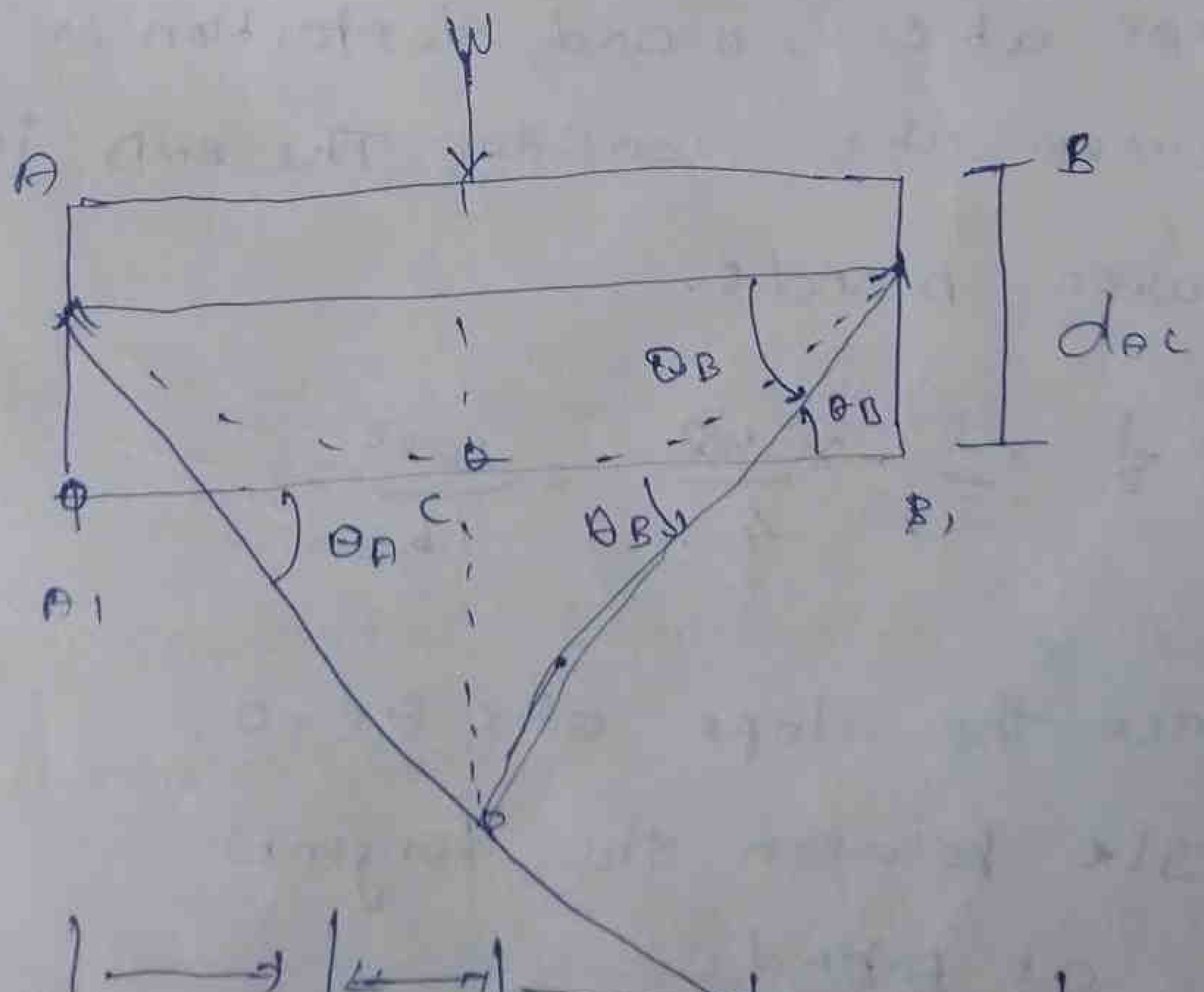
$$\theta_B = \theta_C = \frac{wa^3}{6EI}$$

Deflection

$$Y_B = \frac{wa^2 (4l - a)}{24EI}$$

$$Y_C = \frac{wa^4}{8EI}$$

simply supported beam:



a) Simply Supported Beam

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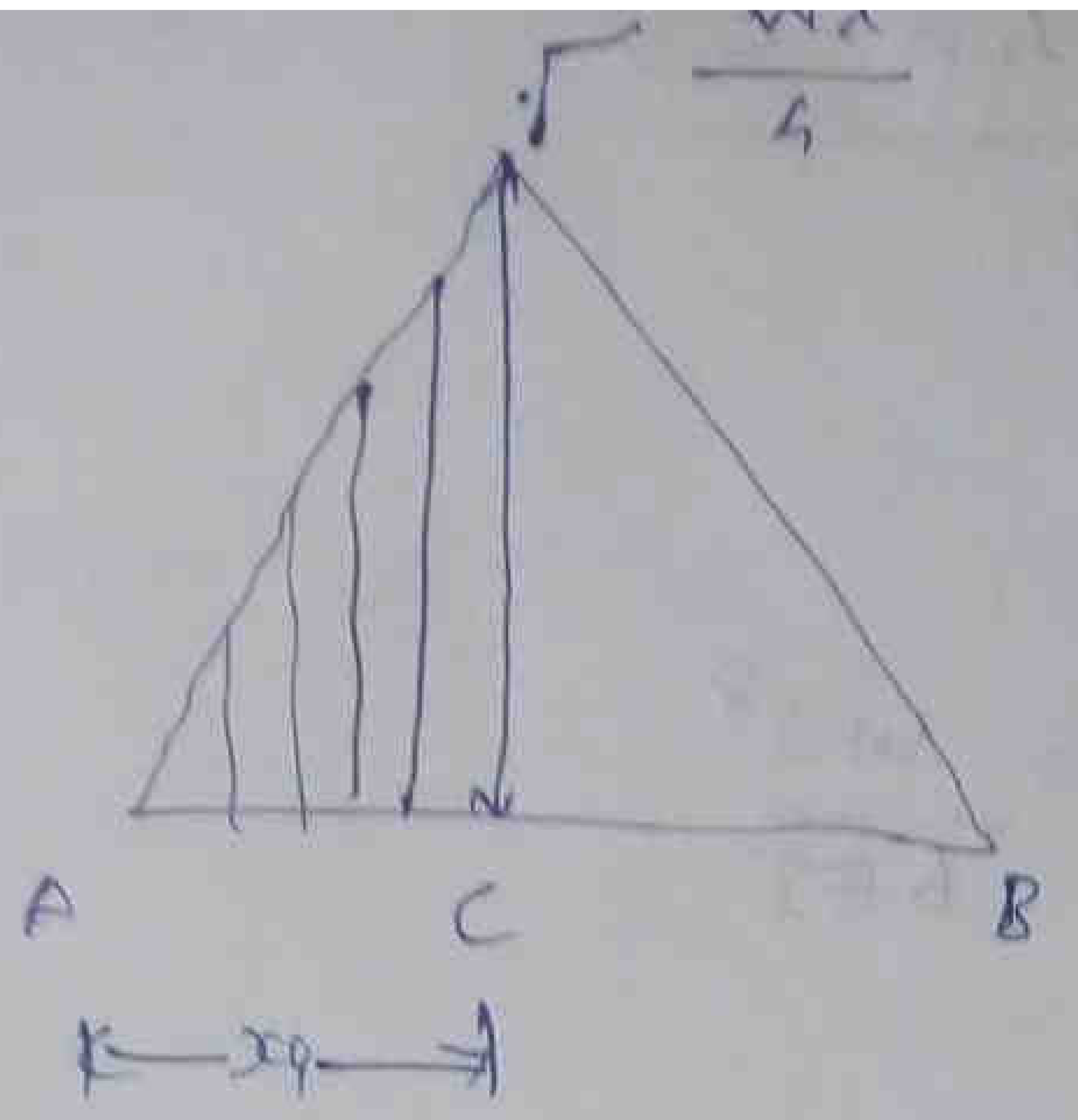


(b) B.M diagram.

Simply supported beam with central concentrated load.

since. loading is Symmetrical slope at A = Slope at B.

Slope at C = 0 and deflection is maximum at C consider The BMD in between A and C.

$$= \frac{1}{2} \times \frac{l}{2} \times \frac{Wl}{4} = \frac{Wl^2}{16}$$

Since the slope at C $\theta_C = 0$.

angle between the tangents at A and C.

$$\theta_{AC} = (\theta_A + \theta_C)$$

$$= \theta_A = \frac{\theta_{AC}}{EI} = \frac{Wl^2}{16EI}$$

Tangential deviation A from the tangent at C.

$$d_{AC} = AA_1 = \frac{a_{AC}\ x_A}{EI}$$

$$= \frac{Wl^2}{16EI} \times \frac{l}{3} = \frac{Wl^3}{48EI}$$

$$\left[\text{since } x_A = \frac{\cancel{2}}{3} \times \frac{l}{\cancel{2}} = \frac{l}{3}\right]$$

But $AA_1 = CC_1$

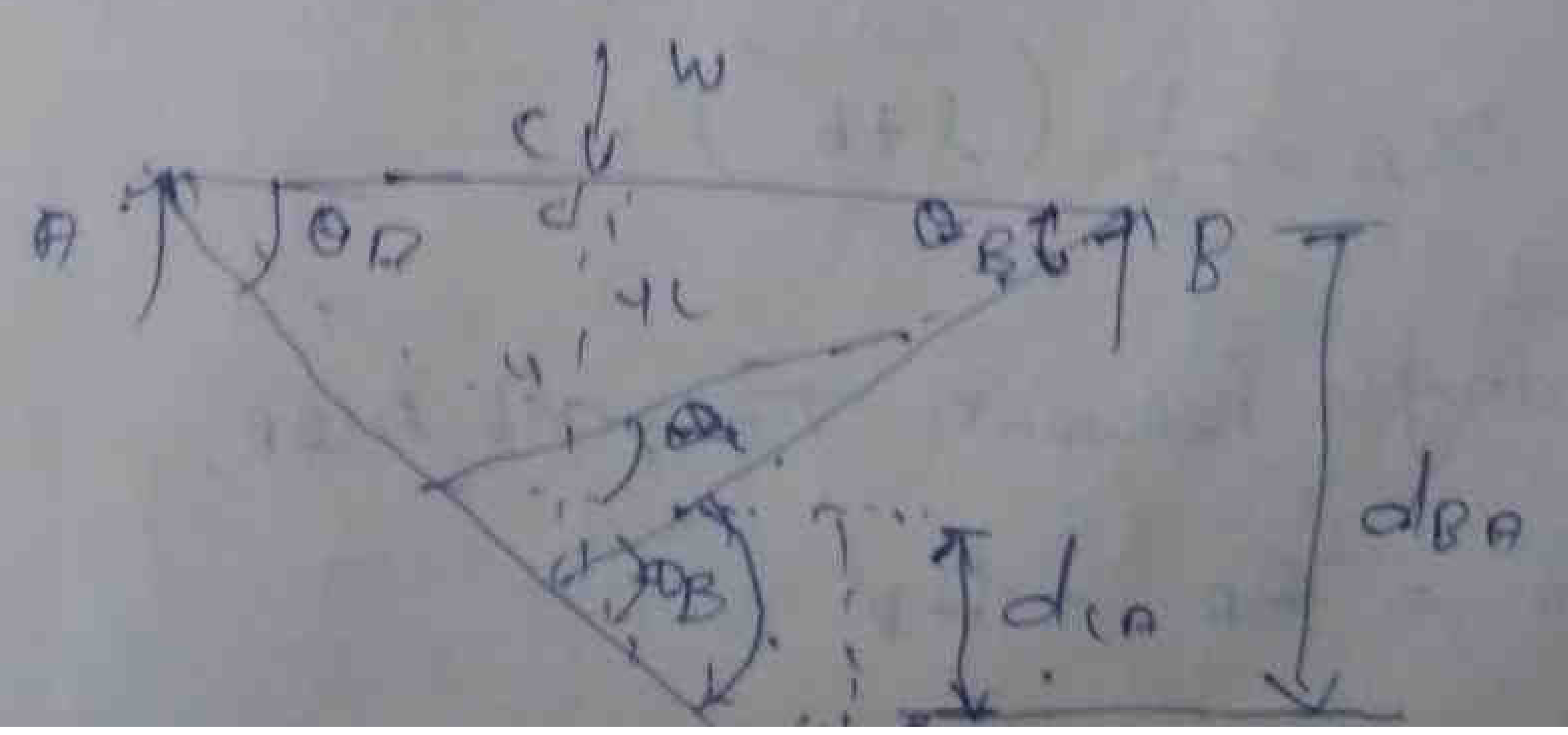
= central deflection y_c

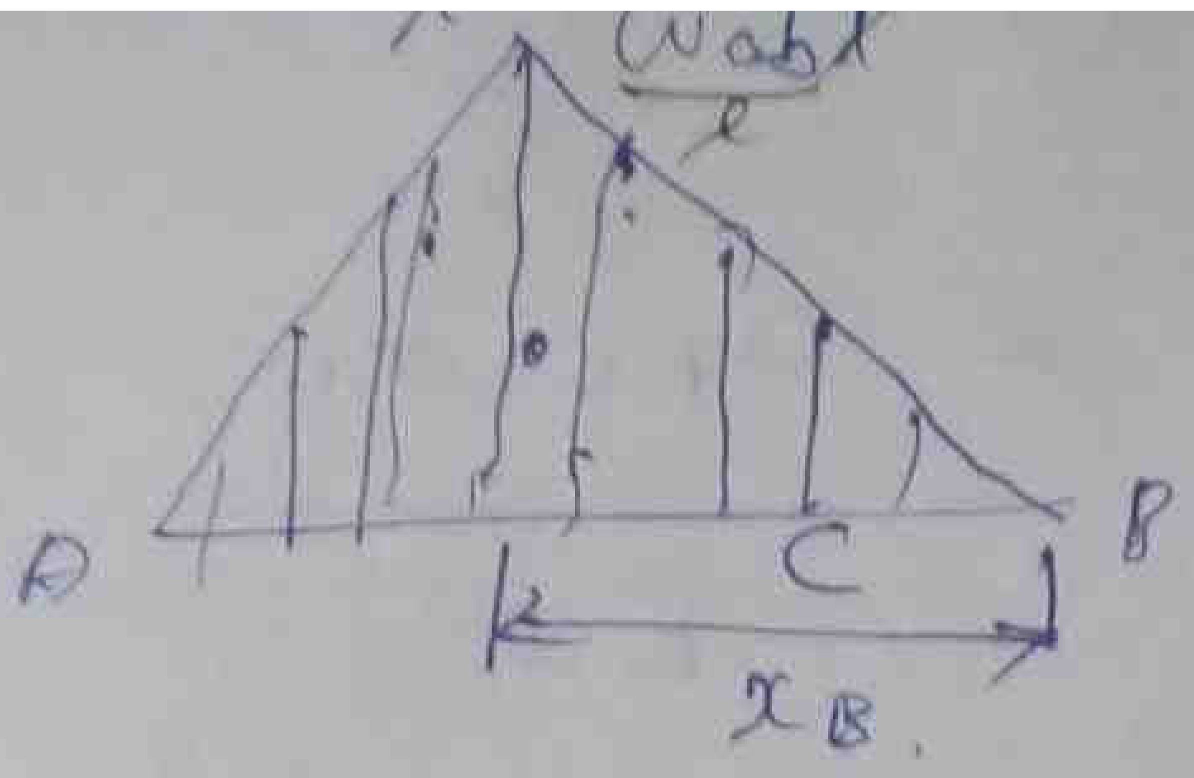
∴ central deflection

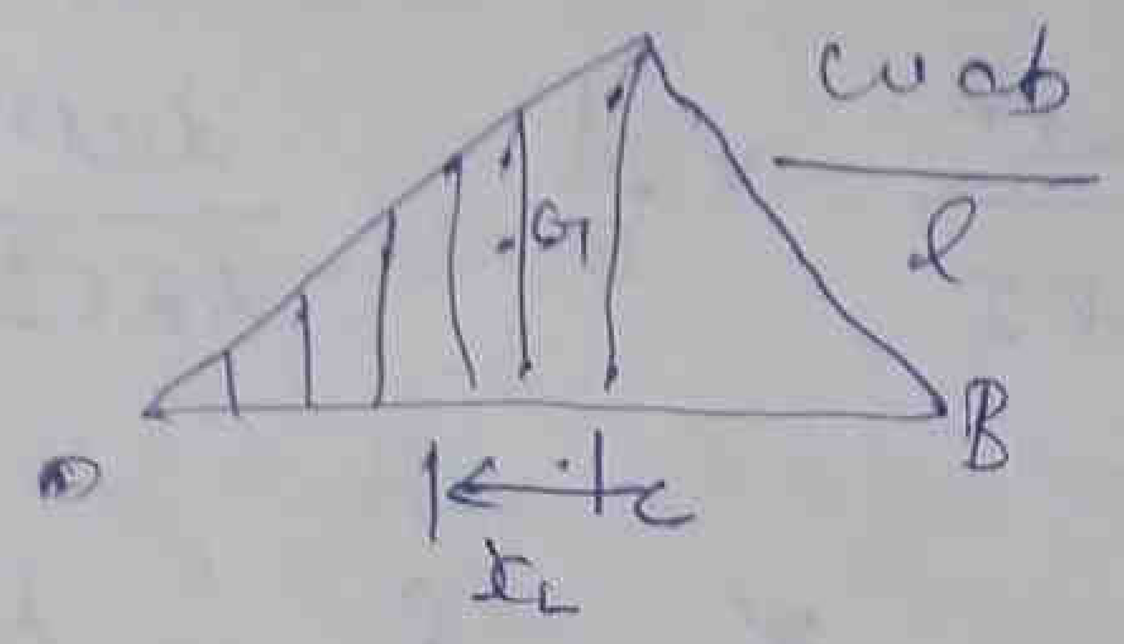
$$y_c = \frac{Wl^3}{48EI}$$

$$\theta_B = \frac{Wl^3}{16EI} \text{ and } y_c = \frac{Wl^3}{48EI}$$

simply supported beam with non centrical concentrated load.







$AC = a$

$CB = b$

Deflection, $y_A = y_B = 0$.

$y_C = (C)$

BMD in b/w A & B.

Area of BMD $= a_{AD}$

$= \frac{1}{2} \times \cancel{l} \times \frac{wab}{\cancel{l}}$

$= \frac{1}{2} wab$

From Centriod

$B = x_B = \frac{1}{3}(l+b)$

The angle between Tangent A & B.

$\theta_{AB} = \theta_A + \theta_B$

$$= \frac{a_{AB}}{EI} = \frac{wab}{2EI} \quad -(1)$$

Tangential deviation of B from the tangent at A.

$$d_{BA} = BB_1 = \frac{a_{AB}\, x_B}{EI}$$

$$= \frac{wab}{2EI} \times \frac{1}{3}(a\ l+b)$$

$$= \frac{wab\,(l+b)}{6EI}$$

But $\tan\theta = \theta_A = \theta_A = \frac{BB_1}{l}$

$$\theta_A = \frac{wab}{6EIl}(l+b) \quad -(2)$$

Substituting in equation (1)

$$\theta_B = \frac{wab}{2EI} - \frac{wab}{6EIl}(l+b)$$

$$= \frac{\cancel{wab}}{12\cancel{EI}l}(\cancel{l+b}).$$

$$= \frac{wab}{6EI}(3l-l-b) - (3)$$

$$= \frac{wab}{6EI}(2l-b) \text{ (or) } \frac{wab}{6EIl}(2l+a).$$

$$\boxed{\text{Since } (l-b) = a} \longrightarrow ③$$

consider of BMD in b/w A & C

Area of BMD $= a_{AC} = \frac{1}{2} \times a \times \frac{wab}{l}$

$$= \frac{wa^2b}{2l}$$

centroid from $C = x_c = \frac{1}{3}a.$

The angle b/w the tangent

$$\theta_{AC} = \theta_A + \theta_c = \frac{a_{AC}}{EI} = \frac{wa^2b}{2EIl} \longrightarrow ④$$

$$\theta_c = \frac{wa^2b}{2EIl} - \theta_A = \frac{wa^2b}{2EIl} = \frac{wab}{6EIl}(l+b)$$

$$= \frac{wab}{6EIl}(3a-l-b)$$

$$= \frac{wab}{6EIl}(3a-a-b-b)$$

$$= \frac{wab}{3EIl}(2a-l-b) \rightarrow ⑤$$

Substituting $(l-b) = a$,

$$\theta_c = \frac{wab}{3EI}(l-2b) \rightarrow ⑥.$$

Substituting $(l-a)$ for b, $\theta_c = \frac{wab}{3EIl}$

$= (2a-l) \rightarrow (7)$

Tangential deviation of C from the tangent at A.

$$d_{CA} = C_1C_2 = \frac{a_{AC} \; x_c}{EI} = \frac{1}{EI} \times \frac{wa^2b}{2l} \times \frac{1}{3}a$$

$$= \frac{wa^3b}{6EIl}.$$

$$CC_2 = a\tan\theta_A = a\,\theta_A$$

$$= \frac{wa^2b^2}{3EIl}(l+b)$$

$$y_c = CC_1 = CC_2 - C_1C_3$$

$$y_c = \frac{wa^2b}{6EIl}(l+b) - \frac{wa^3b}{6EIl}$$

$$= \frac{wa^2b}{6EIl}(l+b-a)$$

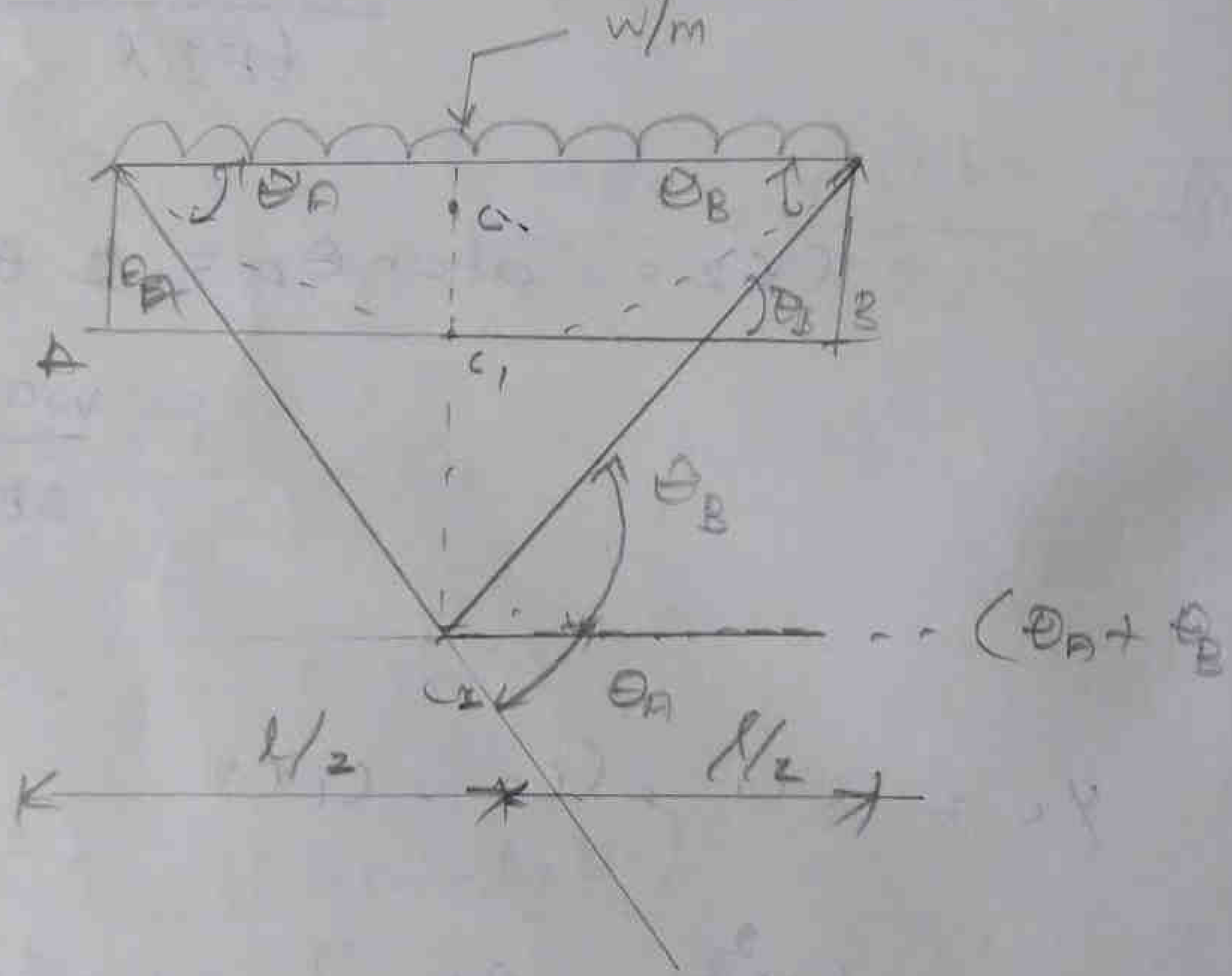
$$= \frac{wa^2b^2}{3EIl}$$

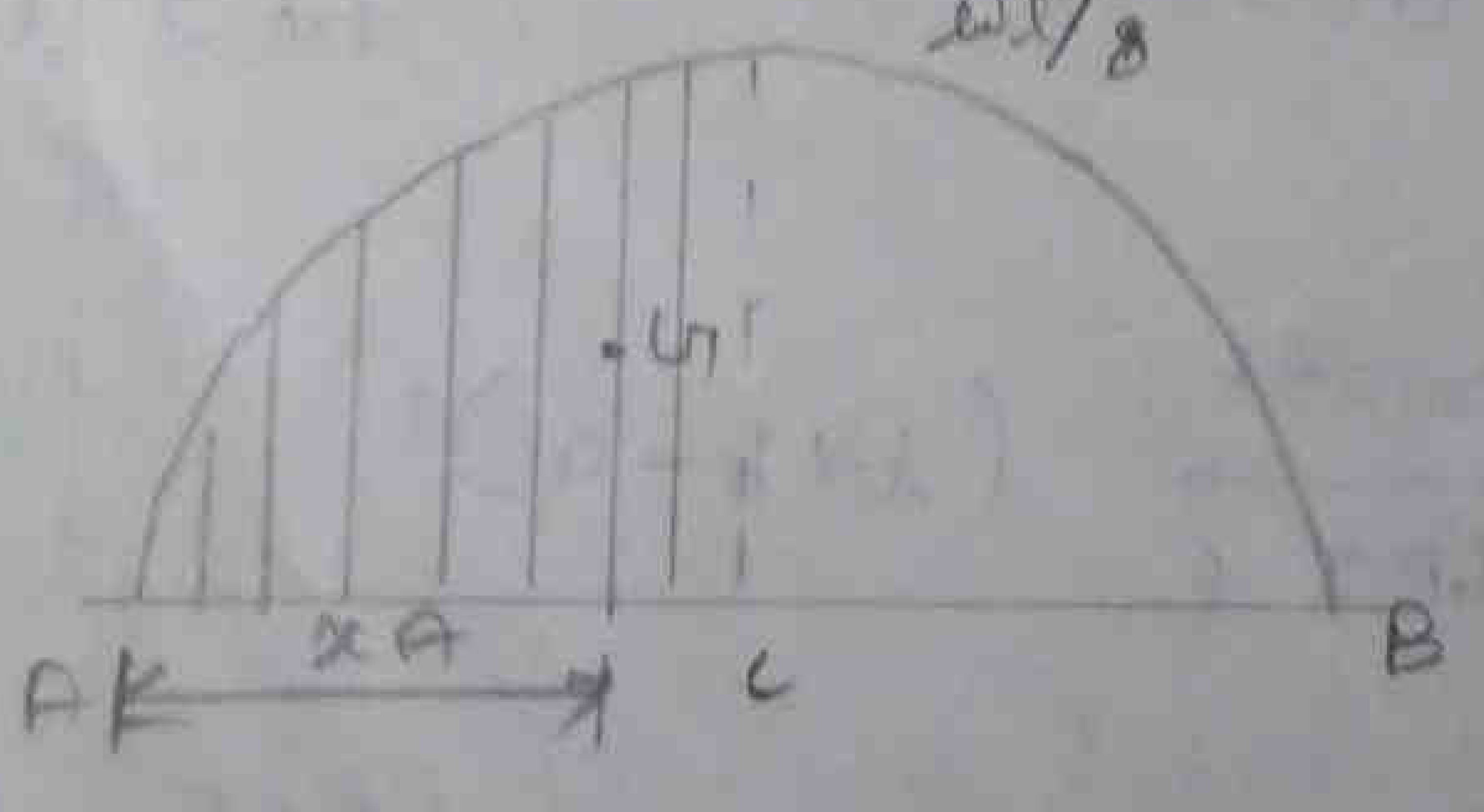
$$\theta_A = \frac{wab}{6EI l}(l+b) \qquad y_A = 0$$

$$\theta_B = \frac{wab}{6EI l}(l+b) \qquad y_B = 0.$$

$$\theta_C = \frac{wab}{3EI l}(a-b) \qquad y_C = \frac{wa^2 b^2}{3EI l}.$$

2) 3.2.4 <u>Simply supported beam with udl throughout</u>





Area of BMD $a_{AC} = \frac{2}{3} \times \frac{l}{2} \times \frac{wl^2}{8}$

$$\boxed{= \frac{wl^3}{24}}$$

The centroid from A.

$$x_A = \frac{5}{8} \times \frac{l}{2}$$

$$\boxed{= \frac{5l}{16}}$$

$$\theta_{AC} = \theta_A + \theta_C = \theta_A$$

$$= \frac{a_{AC}}{EI} = \boxed{\frac{wl^3}{24EI}}$$

$$d_{AC} = AA_1 = CC_1 = y_C$$

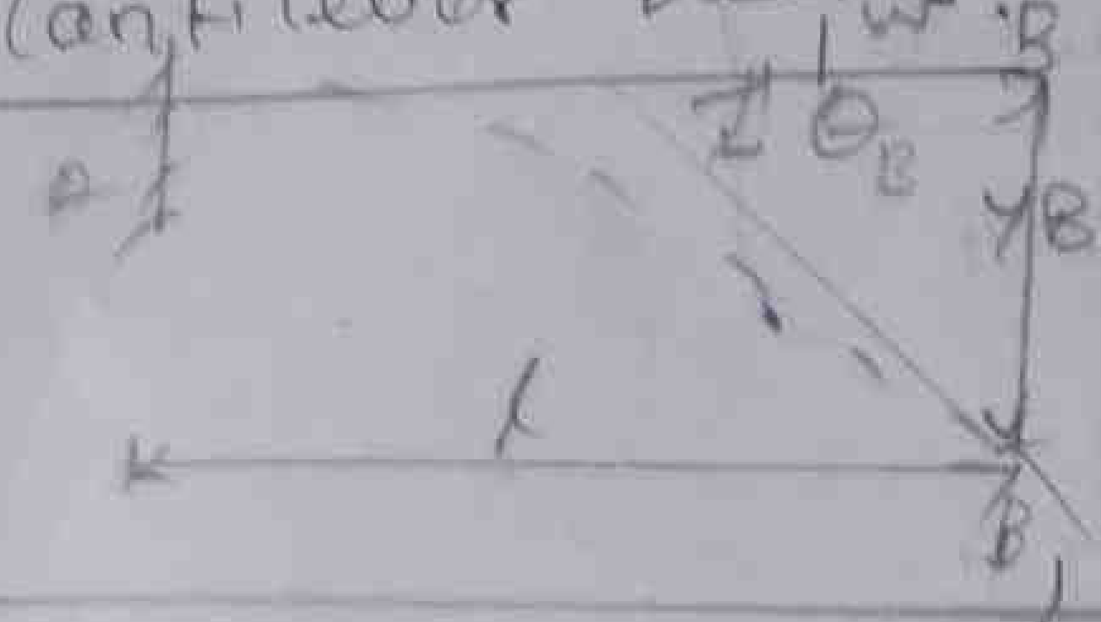
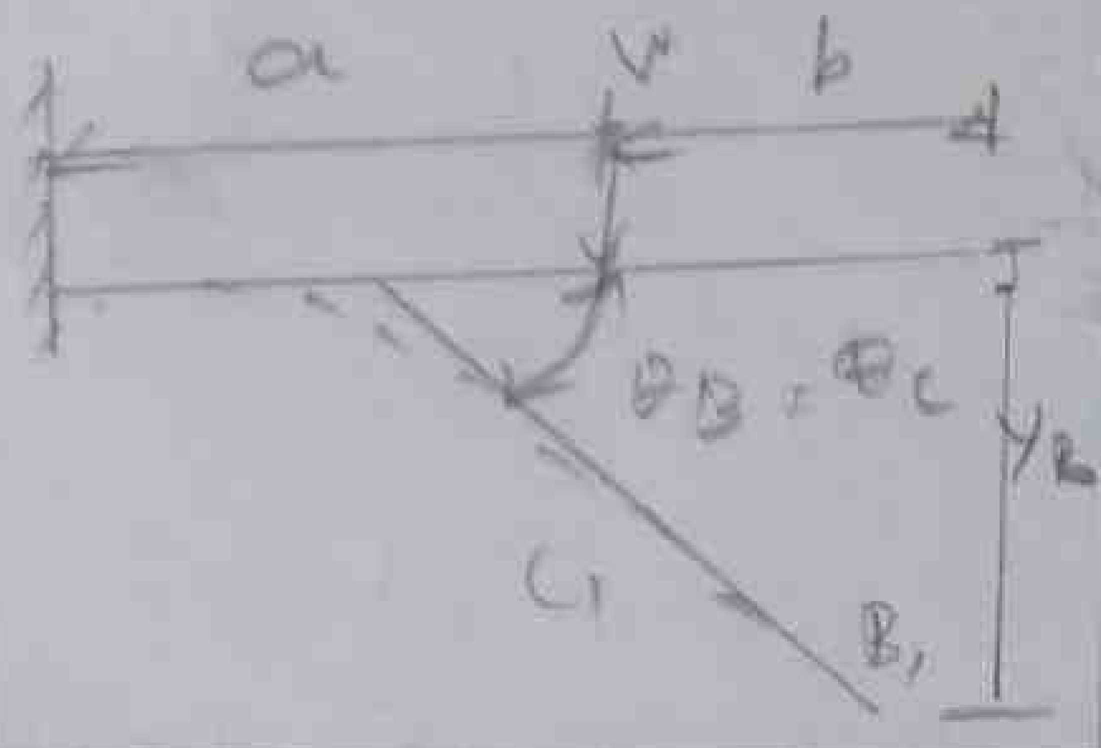
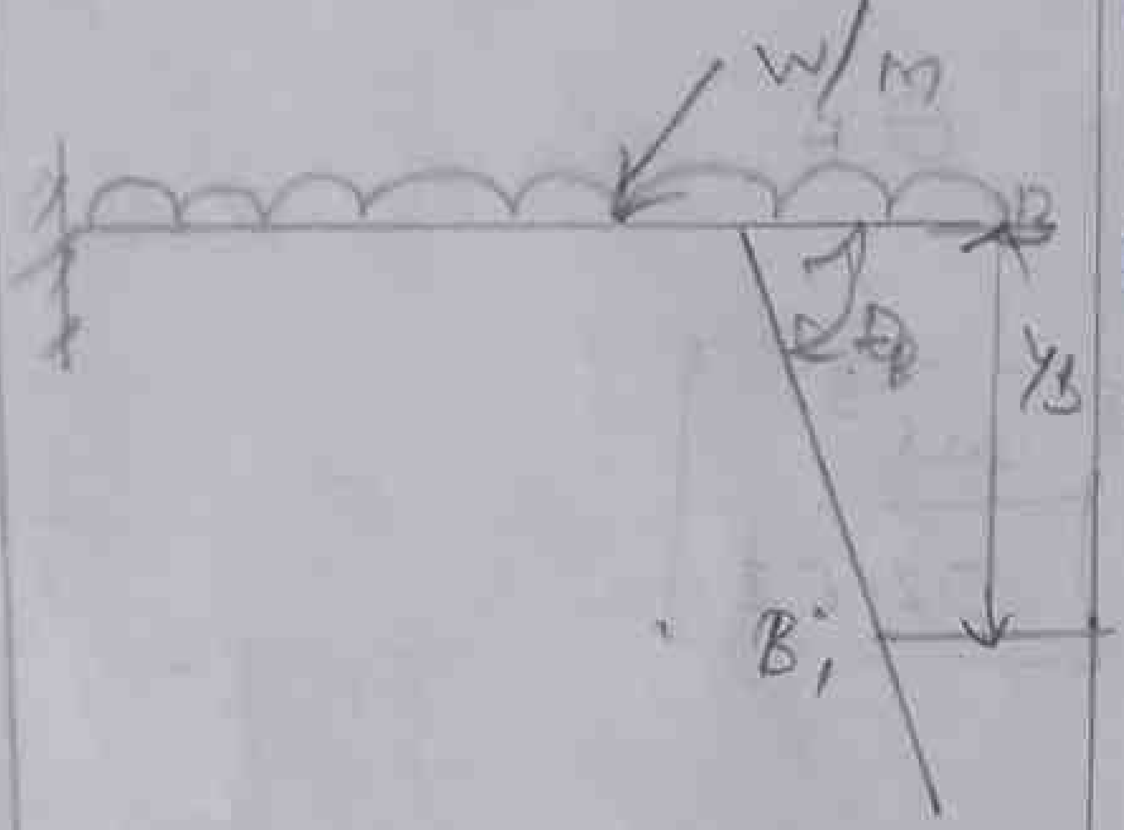
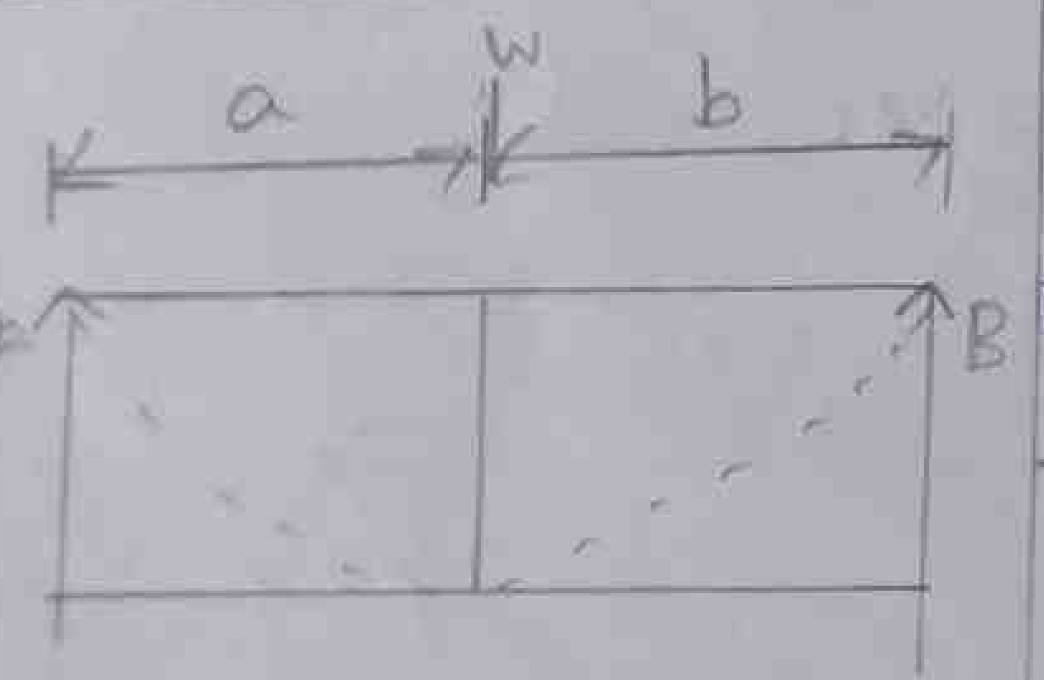
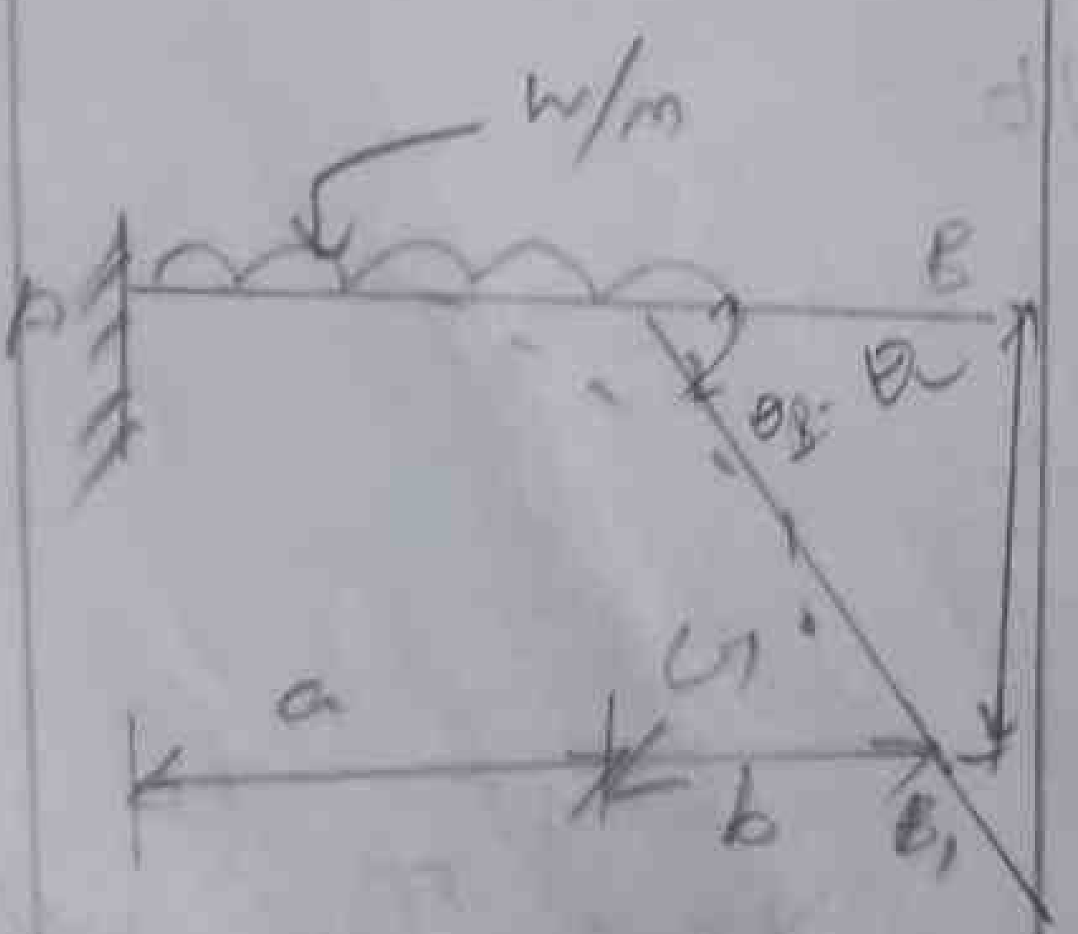
$$= \frac{a_{AC} \cdot x_A}{EI}$$

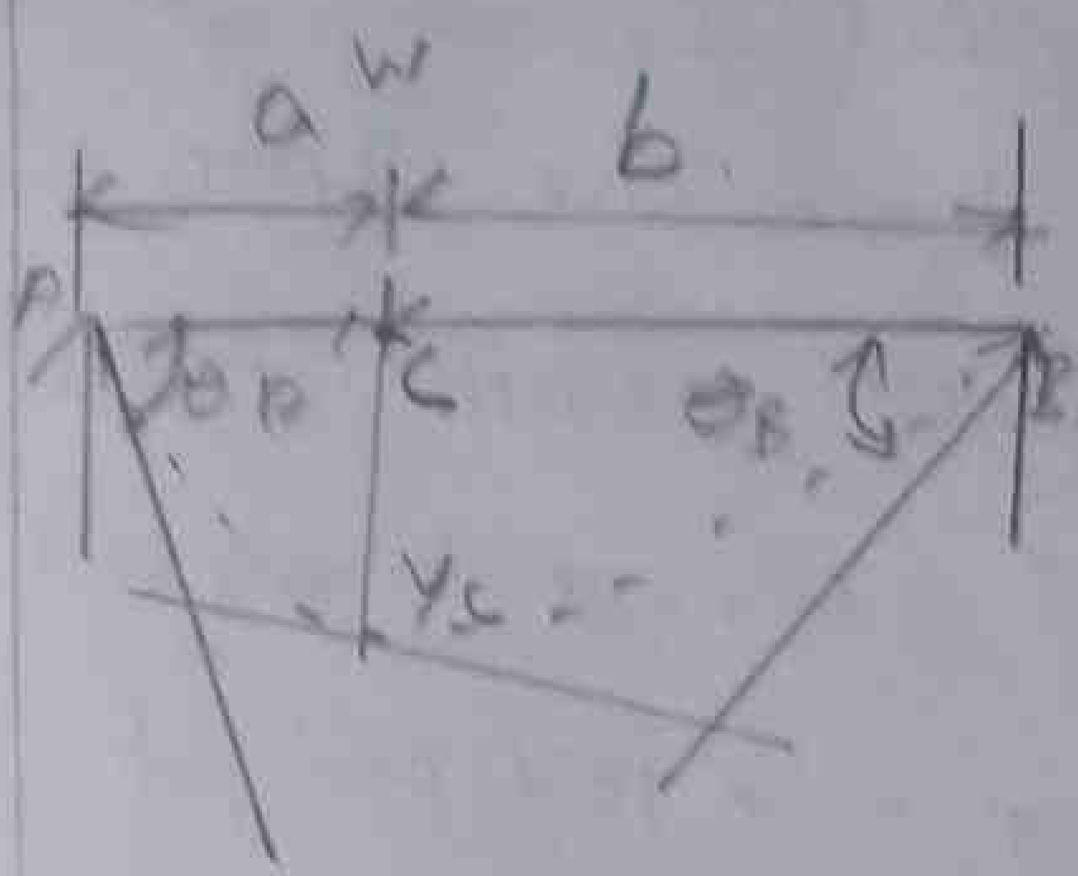
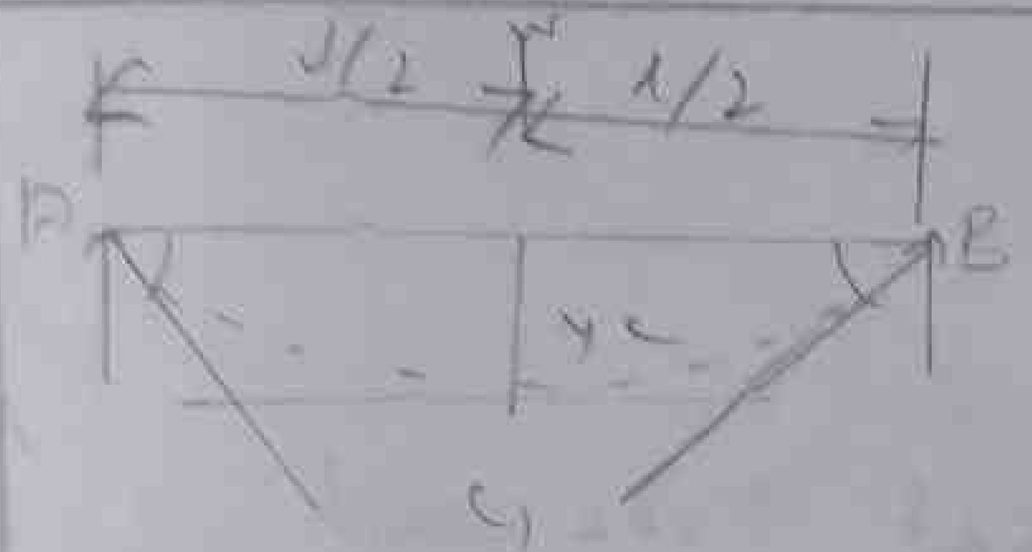
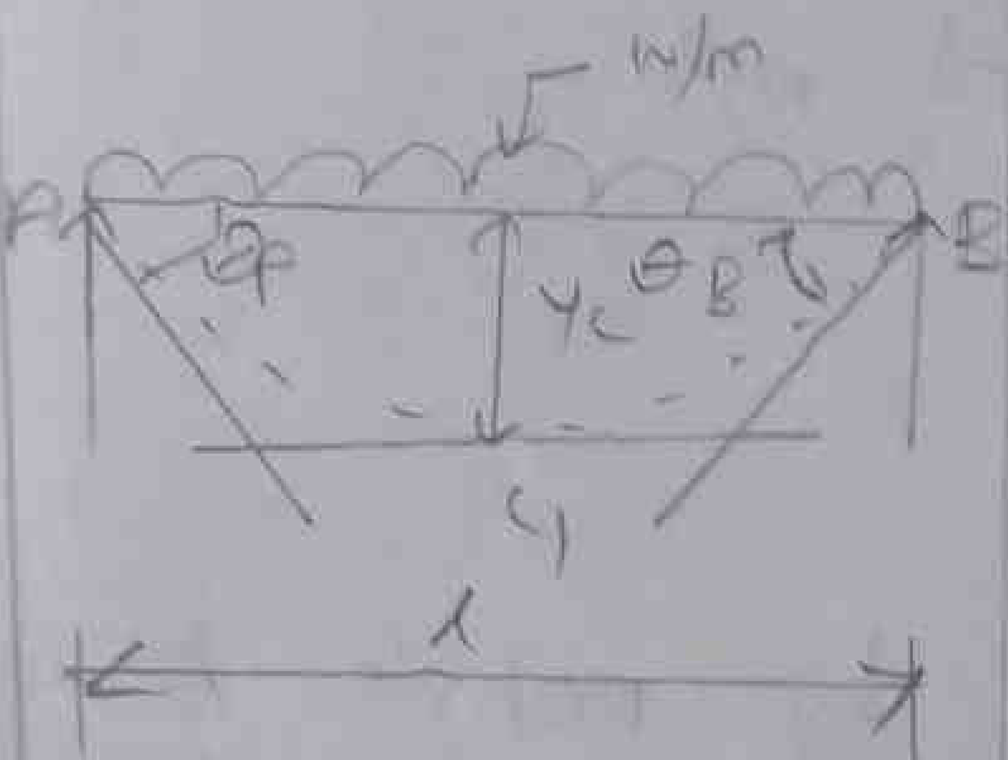
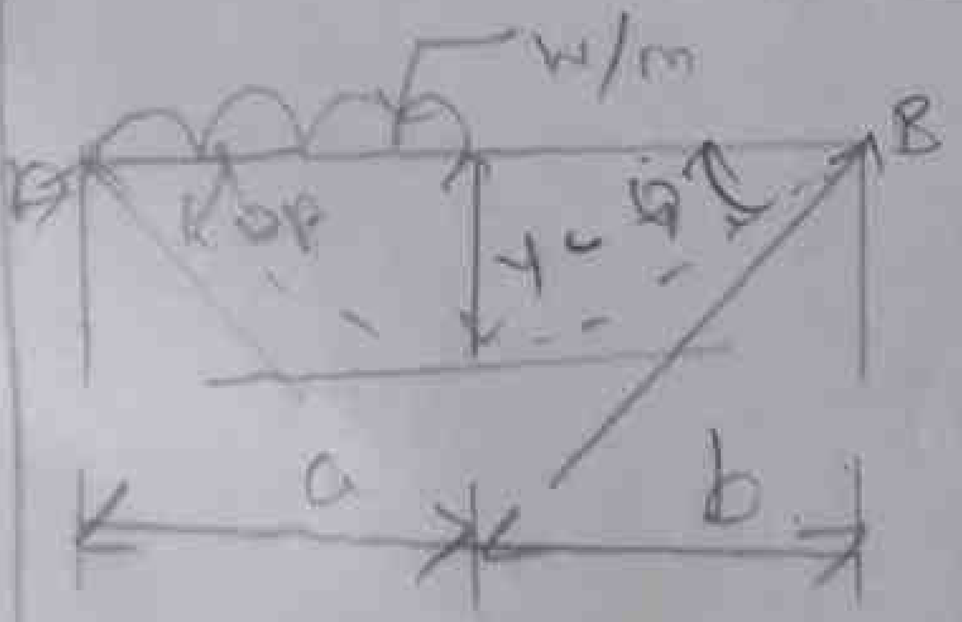
$$= \frac{wl^3}{24EI} \times \frac{5l}{16}$$

$$= \frac{wl^4}{EI} \times \frac{5}{384}$$

$\theta_A = \frac{wl^3}{24EI} = \theta_B$. $y_C = \frac{5}{384} \frac{wl^4}{EI}$.

Slope and deflection.

	Beams	Slope	Deflection
1.	Cantilever beams. 	$\theta_A = 0$ $\theta_B = \frac{wl^2}{2EI}$	$y_A = 0$. $y_B = \frac{wl^3}{3EI}$
2.		$\theta_A = 0$ $\theta_B = \theta_C = \frac{wa^2}{2EI}$	$y_A = 0$. $y_B = \frac{wa^2(3l-a)}{6EI}$ $y_C = \frac{wa^3}{3EI}$
3.		$\theta_A = 0$ $\theta_B = \frac{wl^3}{6EI}$	$y_A = 0$ $y_B = \frac{wl^4}{8EI}$
5.		$\theta_A = 0$ $\theta_B = \theta_C = \frac{wa^3}{6EI}$ $\theta_A = 0$ $\theta_B = \theta_C = \frac{wa^3}{6EI}$	$y_A = 0$ $y_B = \frac{wa^4}{2EI}(4l-a)$ $y_C = \frac{wa^4}{8EI}$
4.		$\theta_A = 0$ $\theta_B = \theta_C = \frac{wa^3}{6EI}$	$y_A = 0$ $y_B = \frac{wa^3}{24EI}(4l-a)$ $y_C = \frac{wa^3}{8EI}$

6.		$\theta_A = \frac{wab}{6EI}(l+b)$ $\theta_B = \frac{wab}{6EIl}(l+a)$ $\theta_C = \frac{wab}{3EIl}(a-b)$	$y_A = 0$ $y_B = 0$ $y_C = \frac{wa^2b^2}{3EIl}$
7		$\theta_A = \theta_B = \frac{wl^3}{16EI}$ $\theta_C = 0$	$y_A = y_B = 0.$ $y_C = \frac{wl^3}{48EI}$
8.		$\theta_A = \theta_B = \frac{wl^3}{24EI}$ $\theta_C = 0.$	$y_A = y_B = 0.$ $y_C = \frac{5}{384}\frac{wl^4}{EI}$
9		$\theta_A = \frac{wa^2}{24EI}(l+b)^2$ $\theta_B = \frac{wa^2}{24EIl}(2l^2-a^2)$	$y_A = y_B = 0.$ $y_C = \frac{wa^2b}{24EIl}(l+3b)$

1). Calculate the slope and deflection at the free end of cantilever beam. of span 3m carrying a point load of 20 kN at the free end by most mohr's area moment method. $E = 2\times10^{11}$ pascal, $I = 2\times10^{-4}$ pascal

Given data:

Cantilever beam.

span, $l = 3m$.

point load, $w = 20kN$ at free end.

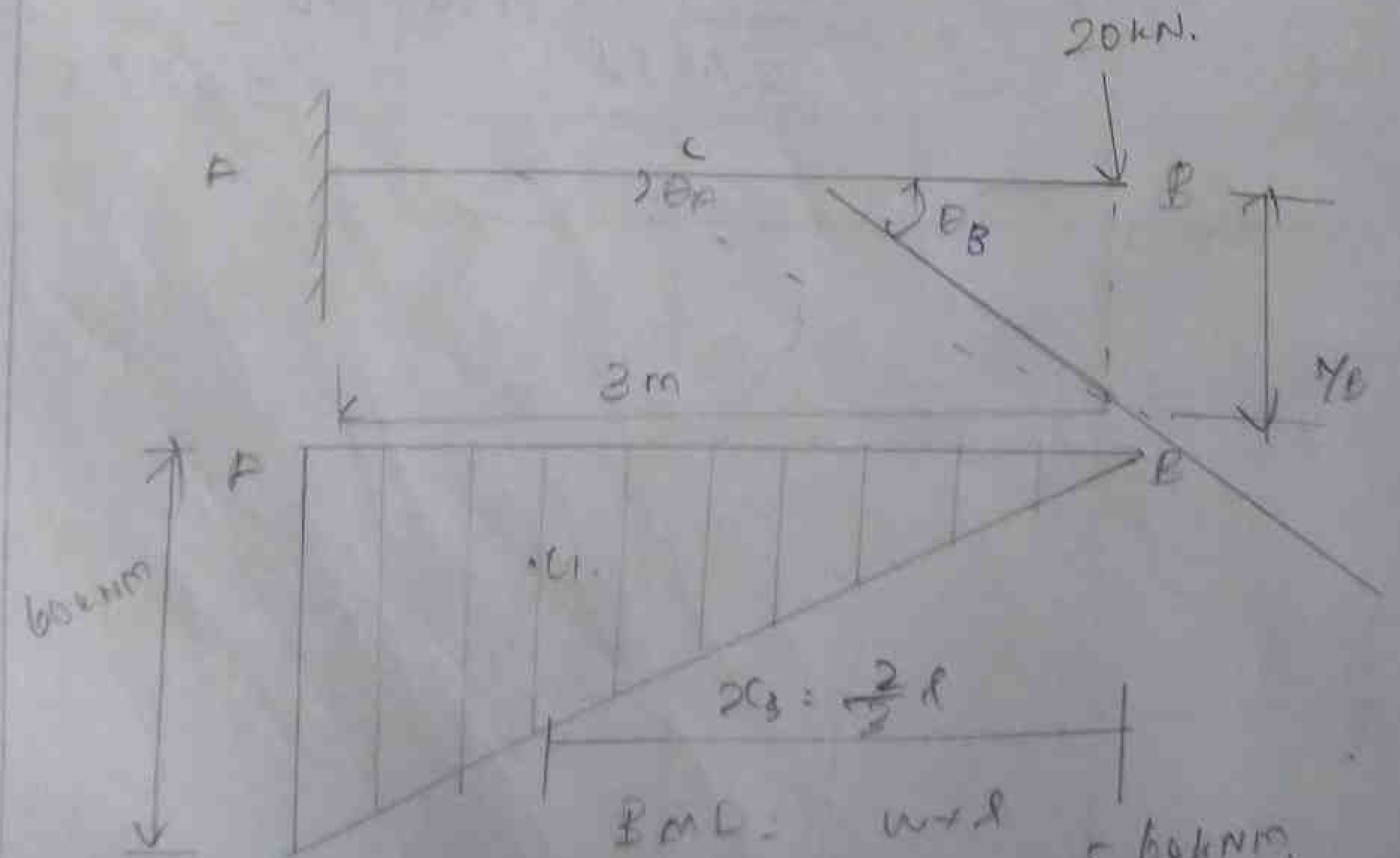
$E = 2\times10^{11}$ pascal $= 2\times10^{11} N/m^2$ $2\times10^{8} kN/m^2$

$I = 2\times10^{-4} m^4 = 2\times10^{-4} m^4$

To find:

Slope and deflection by mohr's Area moment method.

Solution:



BMC = $w \times l$ = 60kNm.

= 20×3

BMD:

BM at $B = 0$.

BM at $A = w \times l$

$= 20 \times 3$

$= 60 \text{ kNm}$.

Area of BMD b/w A & B.

$$a_{AB} = \frac{1}{2} bh.$$

$$= \frac{1}{2} \times 3^{m} \times 60 \text{ kNm}$$

$$= \boxed{90 \text{ m}^2}$$

From centroid B, $x_B = \frac{2}{3} l$.

$$x_B = \frac{2}{\cancel{3}} \times \cancel{3}$$

$$\boxed{x_B = 2 \text{ m.}}$$

$$EI = (2 \times 10^8) \times (2 \times 10^{-4})$$

$$= \boxed{4 \times 10^4} \text{ kN/m}^2 \times \text{m}^4 \; 2 \; (\text{kNm}^2)$$

Slope at A, $\theta_A = 0$.

Slope at B, $\theta_B = a_{AB}$.

$$\theta_B = \frac{a_{AB}}{EI}$$

$$= \frac{90 \text{ kNm}}{4\times10^4 \text{ kNm}^2}$$

$= \cancel{2.45 \text{ kNm}}$.

$= 2250$

$= \boxed{2.250\times10^{-3} \text{ radians.}}$

Deflection at B, $y_B = \frac{a_{AB}}{EI} \times x_B$.

$$= \frac{90}{4\times10^4} \times 2.$$

$y_B = \boxed{4.5\times10^{-3} \text{mm.}}$

Slope at B $\theta_B = \boxed{2.250\times10^{-3} \text{ radians}}$

deflection at B, y_B and d_{AB}. $= \boxed{4.5\times10^{-3}\text{mm}}$

RESULT:

$\boxed{y_B = 4.5\times10^{-3} \text{mm}}$

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22.3.22 ② A Cantilever of length 2l its carrying a load of w at the free end and another load of w at its centre. determine the slope and deflection at free end.

Given data:

span, l = 2l   load @ centre from free end = w.

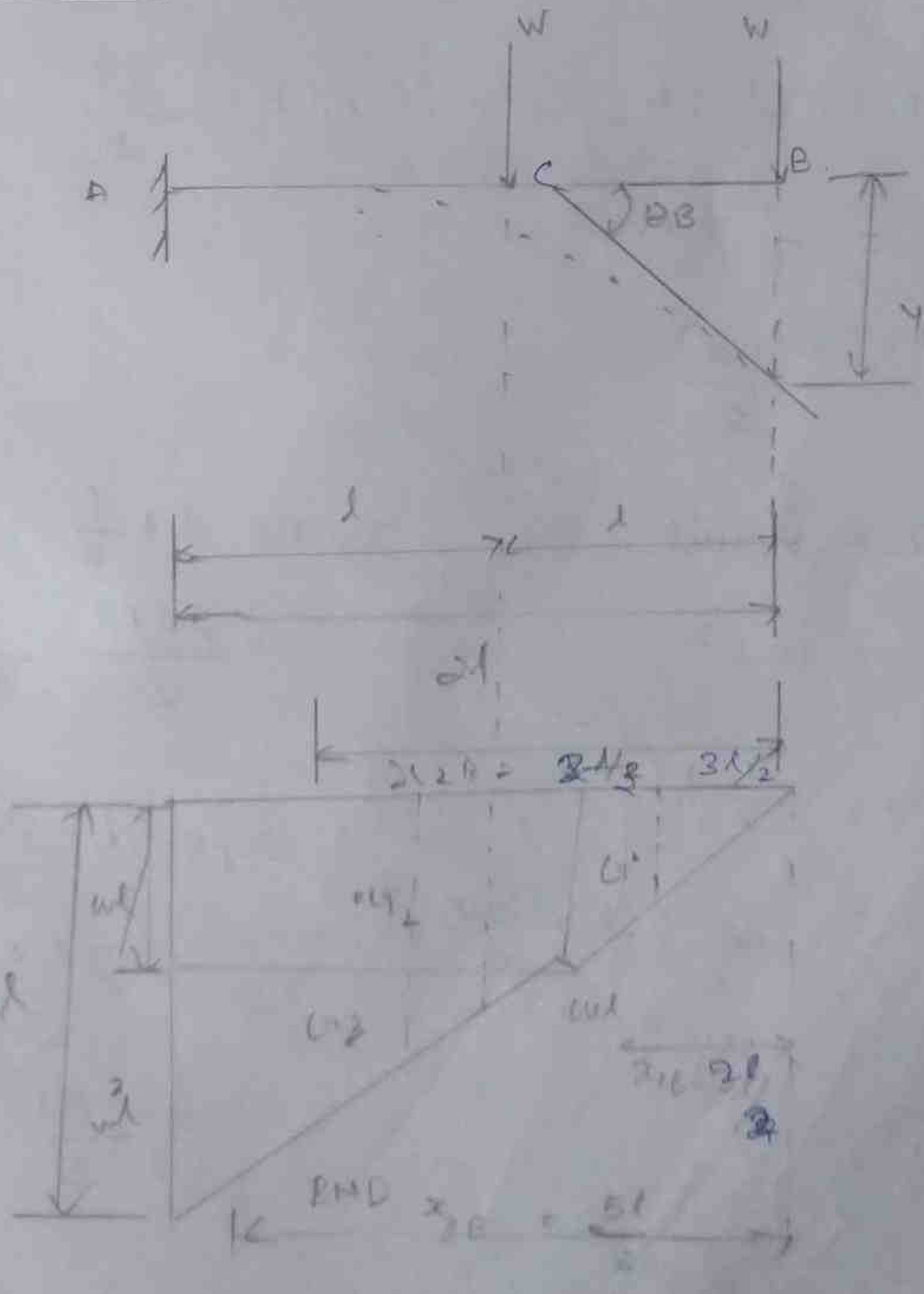
load @ Free = W.

To Find:

Slope and deflection at free end.

BMD Calculation.

Solution:



Solution:

BMD Calculation:

BM @ B : 0

BM @ C = $-wl$

BM @ A : $-(w \times 2l) - (wl)$

$= \boxed{-3wl}$

Area OF BMD

$a_{AB} = a_1 + a_2 + a_3$ centroid from B.

$a_1 = \frac{1}{2} bh$ $\quad x_1 B = \frac{2l}{3}$

$= \frac{1}{2} \times l \times wl$

$= \frac{wl^2}{2}$

$a_2 = l \times wl$ $\quad x_2 B = l + \frac{l}{2}$

$= wl^2$ $\quad = \frac{2l+l}{2} = \frac{3l}{2}$

$a_3 = \frac{1}{2} bh$ $\quad x_3 B = l + \frac{2l}{3}$

$= \frac{1}{2} \times l \times 2wl$ $\quad = \frac{3l+2l}{3}$

$= wl^2$ $\quad = \frac{5l}{3}$

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slope at Free end.

$$\theta_B = \frac{a_{AB}}{EI} = \frac{1}{EI}(a_1 + a_2 + a_3)$$

$$= \frac{1}{EI}\left(\frac{wl^2}{2} + wl^2 + wl^2\right)$$

$$= \frac{1}{EI}\left(\frac{wl^2 + 2wl^2 + 2wl^2}{2}\right)$$

$$\theta_B = \frac{1}{EI}\left(\frac{5wl^2}{2EI}\right)$$

Deflection at Free end, y_B

$$y_B = \frac{a_{AB}}{EI} \times x_B$$

$$= \frac{1}{EI}(a_1 x_1 B + a_2 x_2 B + a_3 x_3 B)$$

$$= \frac{1}{EI}\left[\left(\frac{wl^2}{2} \times \frac{2l}{3}\right) + \left(wl^2 \times \frac{3l}{2}\right) + \left(wl^2 \times \frac{5l}{3}\right)\right]$$

$$= \frac{1}{EI}\left(\frac{2wl^3}{6} + \frac{3wl^3}{2} + \frac{5wl^3}{3}\right)$$

$$= \frac{1}{EI}\frac{wl^3}{EI}\left[\frac{2}{6} + \frac{3}{2} + \frac{5}{3}\right]$$

$$= \frac{wl^3}{EI}\left[\frac{2+9+10}{6}\right]$$

$$= \frac{wl^3}{6EI}\left[2+9+10\right)$$

$$= \frac{wl^3}{6EI} = \frac{\cancel{21}^{7}\, wl^3}{2\cancel{6}EI}$$

$$\boxed{y_B = \frac{7\,wl^3}{2EI}}$$

Slope, $\theta_B : \frac{5wl^2}{2EI}$

deflection, $y_B : \frac{7\,wl^3}{2EI}$

RESULT :

$$= \frac{wl^3}{EI} = \frac{\cancel{21}^{7}\, wl^3}{2\cancel{6}EI}$$

$$\boxed{y_B = \frac{7wl^3}{2EI}}$$

(13) A cantilever beam of span 4 meter its carrying a concentrated load of 15 kN at 3m from the fixed end. Calculate the slope and deflection under the load point and at the free end. $EI = 5\times10^4$ kNm.

Given data:

span = 4m.

load, w = 15 kN.

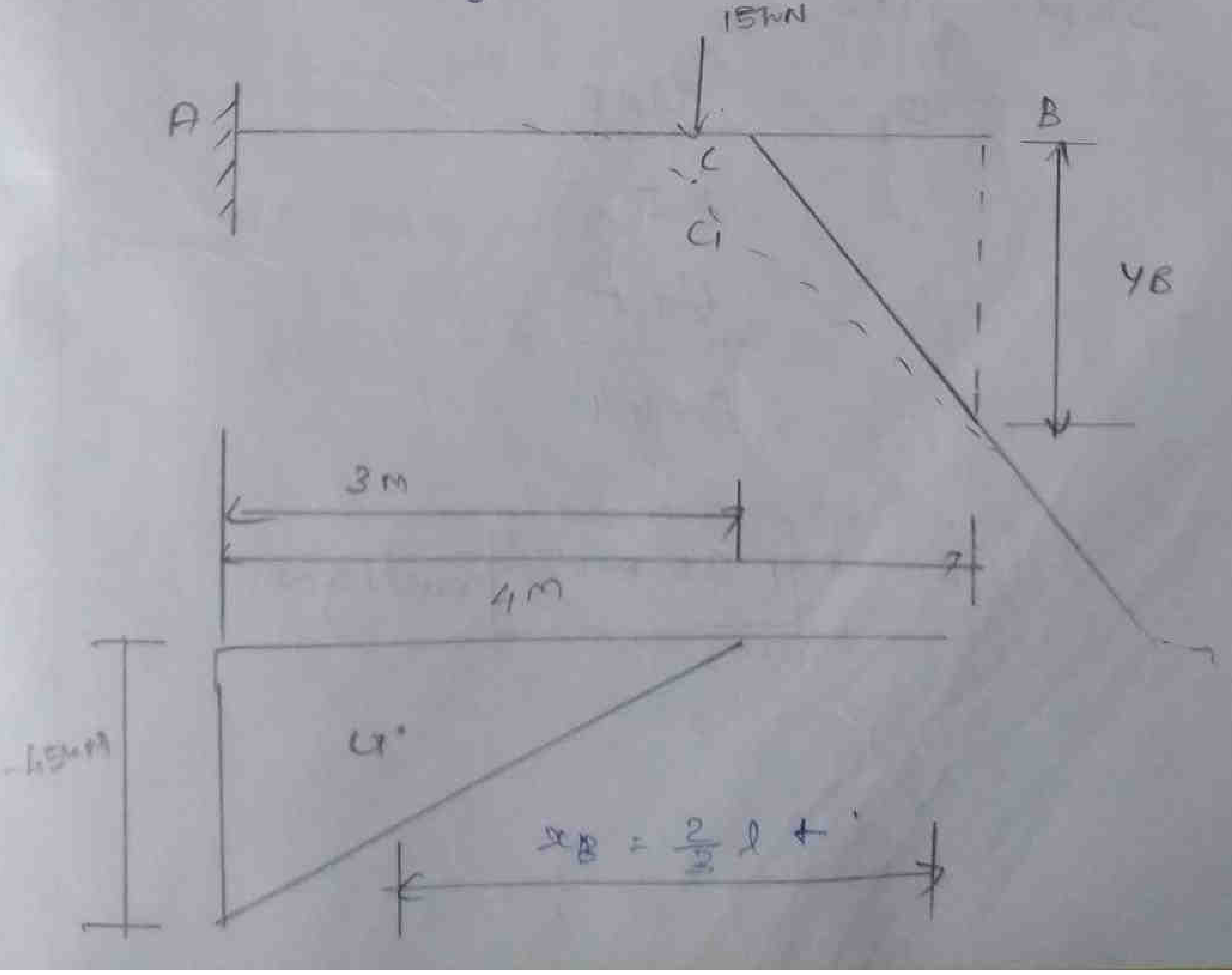
From the fixed end = 3m.

$EI = 5\times10^4$ kNm.

To find:

slope and deflection and BMD calculation.

Solution: Diagram



Solution:

BMD

B.M @ B = 0

B.M @ C = 0.

B.M @ A = $-w \times l$

$= -15 \times 3$

$\boxed{= -45 \text{ kNm}}$

Area of BMD, $a_{AB} = \frac{1}{2} bh$

$= \frac{1}{2} \times 3 \times 45$ kNm.

$= \boxed{67.5 \cancel{m^2} \text{ kNm}^2}$

~~centroid of~~

slope at B

$$\theta_B = \frac{a_{AB}}{EI}$$

$$= \frac{67.5}{5 \times 10^4}$$

$= \boxed{1.35 \cancel{\times 10}^{-3}}$ radians.

deflection at y_B =

$$y_B = \frac{a_{AB}}{EI} \times x_B$$

$$x_B = \left(\frac{2}{3} l\right) + 1$$

$$= \left(\frac{2}{\cancel{3}} \times \cancel{3}\right) + 1$$

$$= 2 + 1$$

$$\boxed{x_B = 3.}$$

$$y_B = \frac{67.5}{5 \times 10^4} \times 3$$

$$y_B = 4.05 \times 10^{-3}\ m \quad \boxed{y_B = 4.05\ mm}$$

Area of BMD at AC.

$$a_{AC} = \frac{1}{2} bh$$

$$= \frac{1}{2} \times 3 \times 45$$

$$= \boxed{67.5\ kNm^2}$$

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Slope at AC, $\theta_C = \frac{a_{AC}}{EI}$

$= \frac{67.5}{5 \times 10^4}$

$\boxed{\theta_C = 1.35 \text{ radians.}}$

deflection at C, $y_C = \frac{a_{AC}}{EI} \times x_C$

$x_C = \left(\frac{2}{3} \times l\right)$

$= \left(\frac{2}{3} \times 3\right)$

$\boxed{x_C = 2m}$

$y_C = \frac{a_{AC}}{EI} \times x_C$

$= \frac{67.5}{5 \times 10^4} \times 2$

$y_B = 2.7 \times 10^{-3} M$

$\boxed{y_B = 2.7mm}$

slope at A, θ_A $\boxed{= 0.}$

slope at C, θ_C = $\boxed{1.25 \text{ radians}}$

slope at B, θ_B = $\boxed{1.25 \text{ radians.}}$

deflection at A, θ_A = $\boxed{0.}$

deflection at B, θ_C = $\boxed{2.7mm}$

deflection at B, θ_B = $\boxed{4.05mm}$

4) A cantilever of span 5metre carry is a uniformaly distributed load of 15kn/m run over the ~~end~~ enterier span. the cross size of the beam is 200x300mm. Young's modulus is $1.5 \times 10^5 N/mm^2$. Calculate the maximum slope and deflection at the free end of the beam.

Given data:

span, l = 5m

load, w = 15kN/m.

size of the beam = 200 x 300mm.

young's modulus E = $1.5 \times 10^5 N/mm^2$

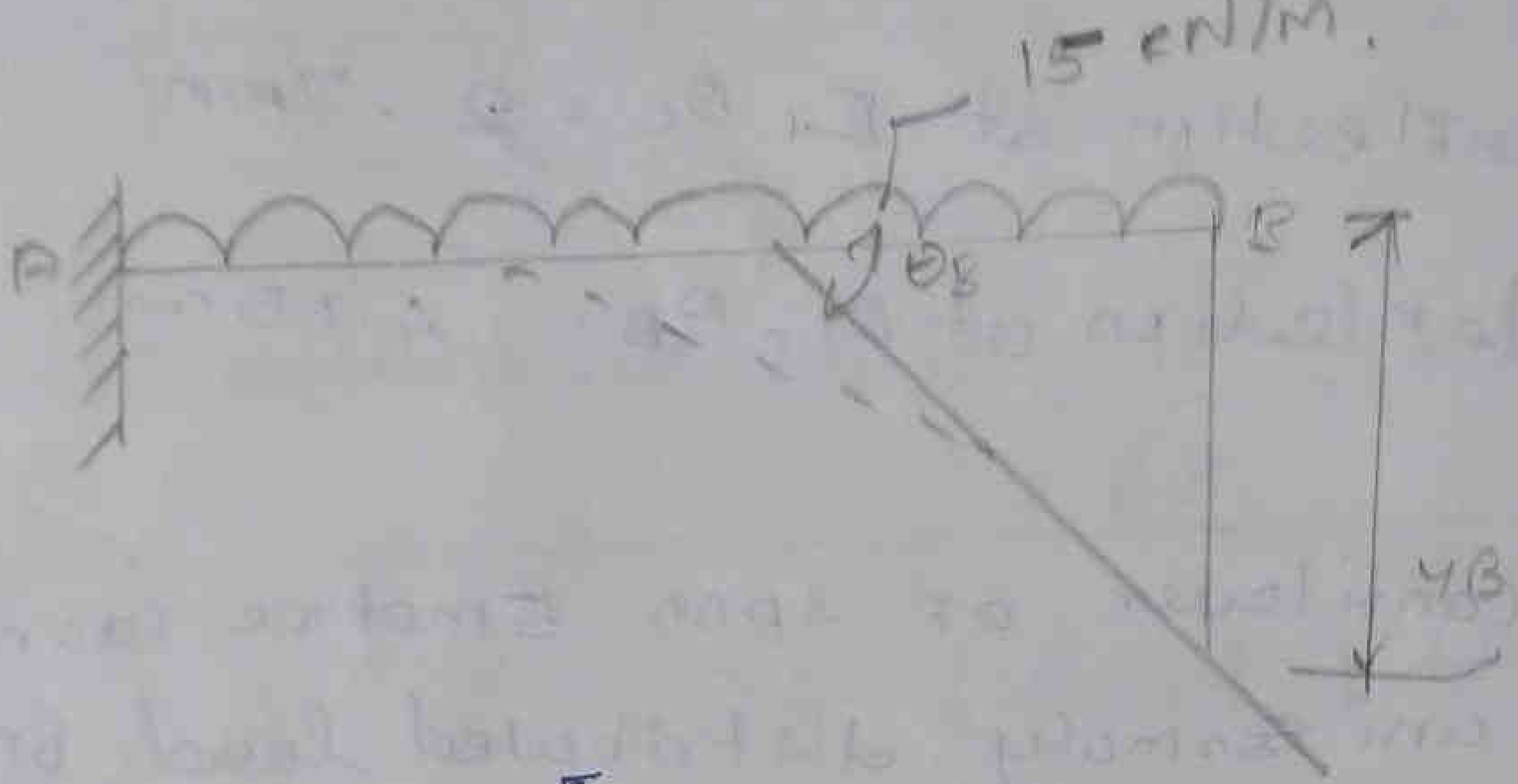
To find:

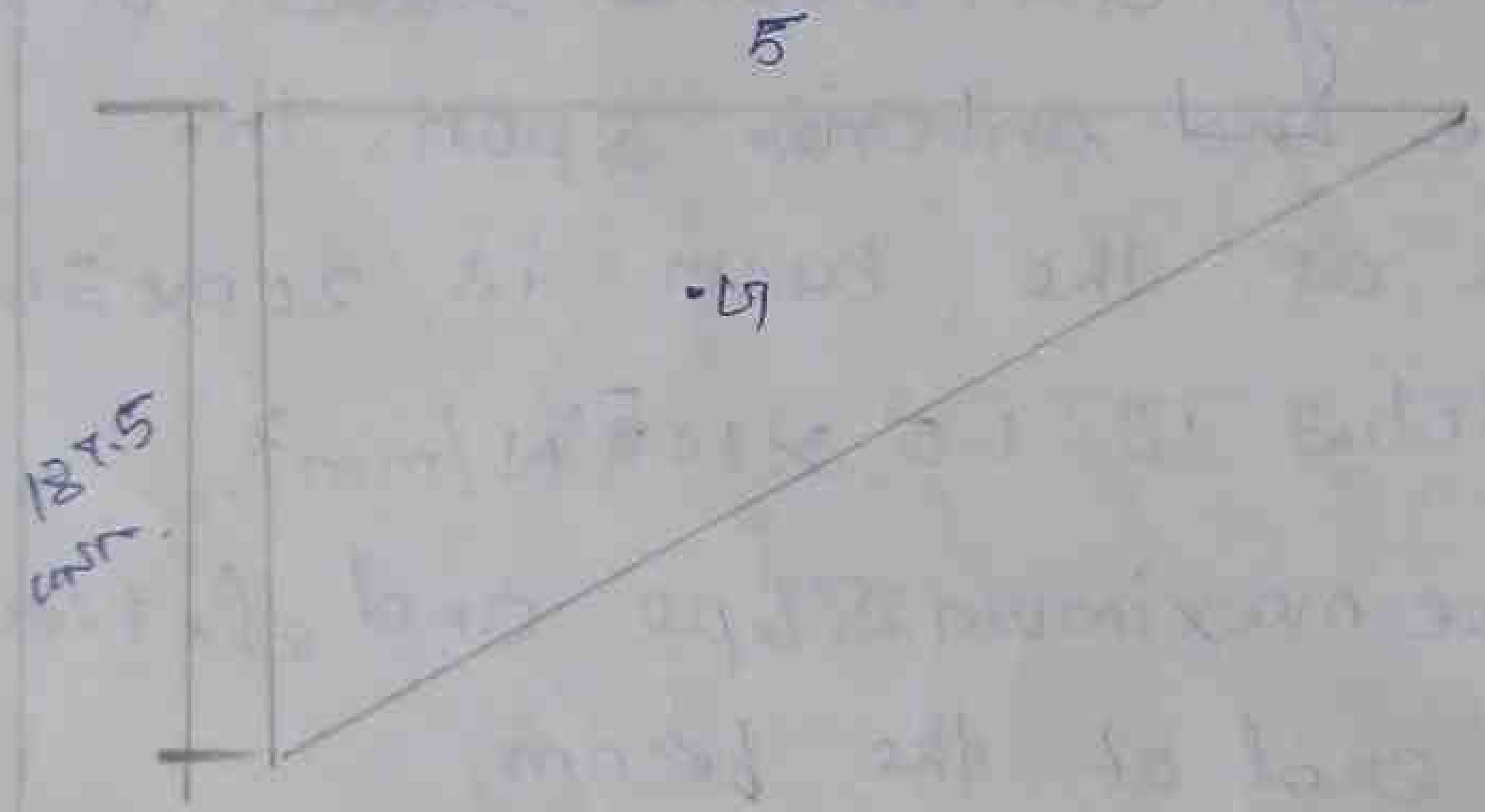
Calculate the max. slope and deflection.

Solution :

moment of Inertia $= \frac{bd^3}{12} \times \frac{200\times800^3}{12}$

$= \underline{4.5\times10^8 \text{ mm}^4}$





BMD

BM at B = 0.

BM at A = $w \times l \times \frac{l}{2}$

$= 15 \times 5 \times \frac{5}{2}.$

$= 187.5$ kNm.

~~Area of AB~~ =

$EI = 1.5 \times 10^5 \times 4.5 \times 10^8$

$= \boxed{6.75 \times 10^{13}\ N/mm^2}$ $\cdot 10^3 = N$ to kN

$= 6.75 \times 10^4\ N/m^2$ $10^6 = mm^2$ to m^2

slope at $\theta_B = \dfrac{a_{AB}}{EI}$

$a_{AB} = \dfrac{1}{3} \times bh$

$= \dfrac{1}{3} \times 5^{m} \times 187.5\ kNm^2$

$\boxed{a_{AB} = 312.5 m^2}$ radians

$\theta_B = \dfrac{a_{AB}}{EI}$

$= \dfrac{312.5}{6.75 \times 10^{4}}$

$= 4.62\ \cancel{\times 10^{-3}}$ r

$= \boxed{4.62 \times 10^{-3}\ \text{radians}}$

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$$y_B = \frac{a_{AB}}{EI} \times x_B$$

$$= \frac{312.5}{}$$

$$x_B = \frac{3}{4} \times l$$

$$= \frac{3}{4} \times 5$$

$$= 3.75$$

$$y_B = \frac{a_{AB}}{EI} \times x_B$$

$$= \frac{312.5}{6.75 \times 10^{\cancel{5}4}} \times 3.75$$

$$= \cancel{1.73}\ 0.0174M$$

Result:

Slope at A, $\theta_{AB} = \boxed{312.5}$

Deflection $y_B = \boxed{0.0174 \cancel{N}M}$

By Direct For mula.

formula:

Slope	Deflection
$\theta_A = 0$	$Y_A = 0$
$\theta_B = \frac{wl^3}{6EI}$	$Y_B = \frac{wl^4}{8EI}$
$= \frac{15 \times 5^3}{6 \times 6.75 \times 10^4}$	$= \frac{15 \times 5^4}{8 \times 6.75 \times 10^4}$
$= \boxed{4.62 \times 10^{-3}}$	$= \cancel{17.3 \times 10^{-3}}$ $= \boxed{\cancel{0.17}}$ M. $= 0.0173$ M

5. A cantilever 1meter long is of rectangular section of width = 40mm and depth = 60mm. A calculate the maximum wall then allowed on the entirer of the cantilever beam without exceeding a deflection of 3.5mm at the free end $E = 7 \times 10^4 N/mm^2$.

Given data:

span, $l = 1\,m$

width, $b = 40mm$

depth, $d = 60mm$

$E = 7\times10^4$ mm.

$y_{max} = 3.5$ mm.

Solution:

$$y_{max} = \frac{wl^4}{8EI}$$

$$\cancel{3.5mm} = \frac{l^4}{8 \cancel{\times 7\times10^4}}$$

$$\cancel{3.5}\, I = \frac{bd^3}{12}$$

$$= \frac{40 \times 60^3}{12}$$

$$\boxed{I = 720\ mm^4}$$

$$y_{max} = \frac{\cancel{3.5}\, wl^4}{8EI}$$

$$= \frac{l^4}{8 \times 7\times10^4}$$

$$= \cancel{122.5\times10^3}\ 14.11$$

$$3.5 = \frac{l^4}{8 \times 50.4\times10^6} = \boxed{14.112\ kN/m}$$

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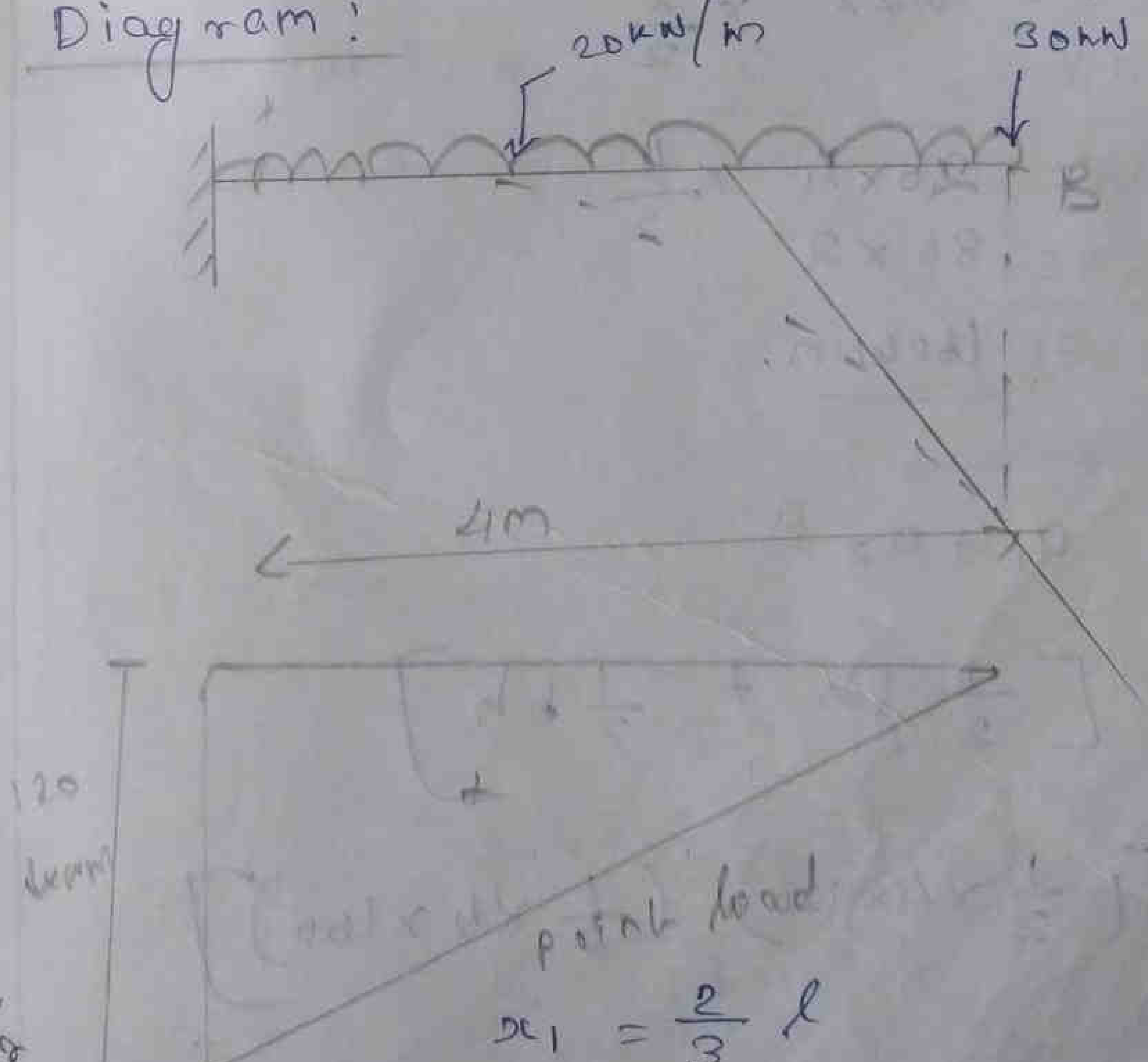
$EI = \cancel{\times} 7 \times 10^4 \times 720$

$= 50.4 \times 10^6$

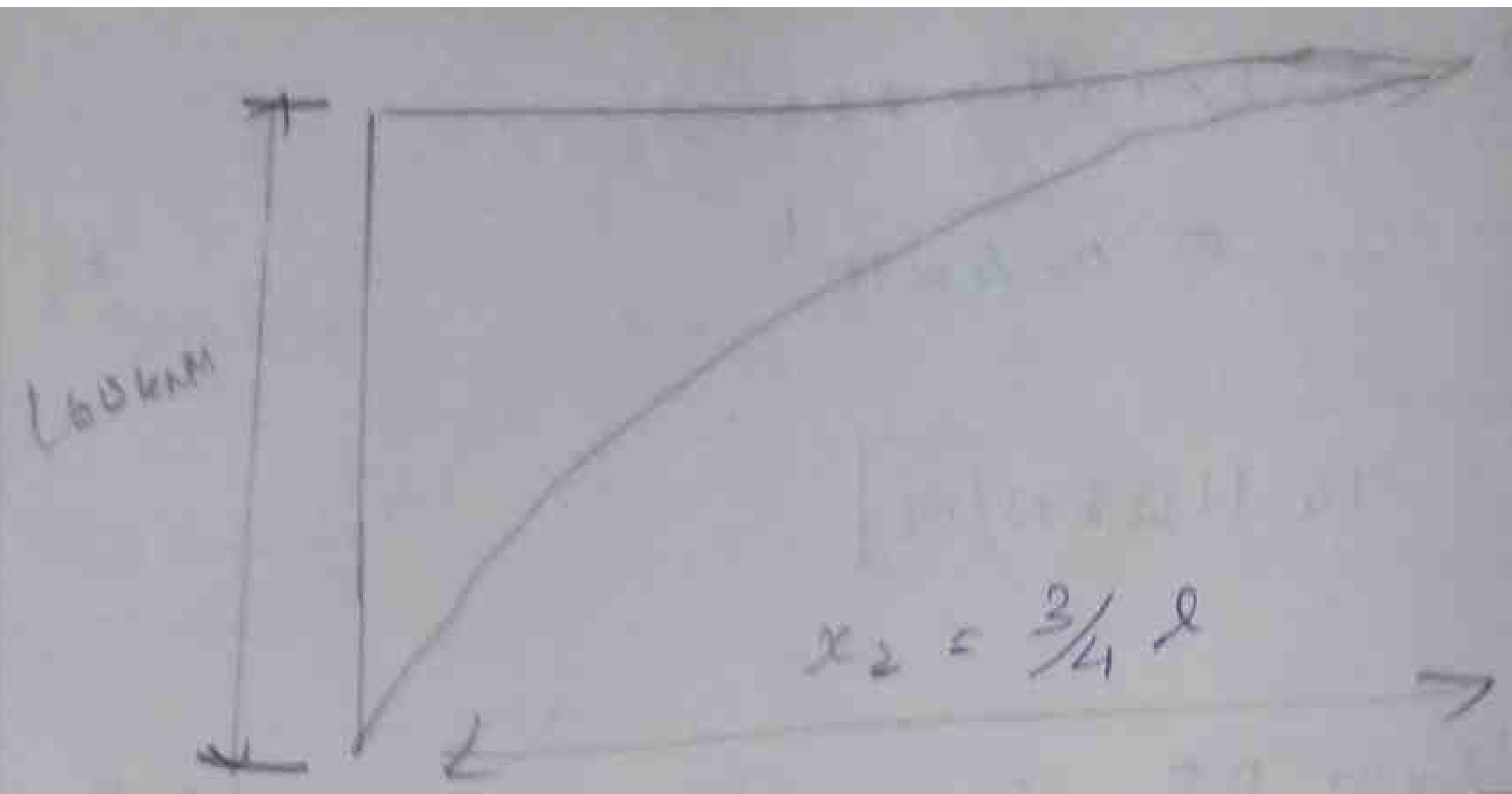
$\boxed{EI = 14.112 kN/m}$

4.3.22 A cantilever of 4m span carrigs is a uniformuly distributed load of 20 kN/m spread over run its enteeor length. in adition flu udl its carry is a concentrated load of 30 kN at the free end. calculate the slope and deflection at the free end. by moment area method. $E = 2 \times 10^5 N/mm^2$, $I = 8 \times 10^7 mm^4$.

Diagram:



$x_1 = \frac{2}{3} l$



$E = (2\times10^5 \times 8\times10^7)$
$= (N/mm^2) \times mm^4$
$= \boxed{1.6\times10^4\ kN/m^2}$

BMD: (point load)

BM at B = 0

BM at A = 30×4 = 120kNM

BMD: (udl)

BM at B = 0

BM at A = $w \times l \times \frac{l}{2}$

$= 20\times4 \times \frac{4}{2}$
$= 80\times2$
$= 160kNM.$

$a_{AB} = a_1 + a_2$

$= \left[\frac{1}{2}bh + \frac{1}{3}bh\right]$

$= \left[\left(\frac{1}{2}\times4\times120\right) + \left(\frac{1}{3}\times4\times160\right)\right]$

$a_{AB_1} = a_1$

$= \left(\frac{1}{2} bh\right) = \frac{1}{2} \times 4 \times 120 = 240\ \text{mm}^2$ kW

$a_{AB_2} = a_2$

$= \left(\frac{1}{3} bh\right) = \frac{1}{3} \times 4 \times 160 = 213.33\ \text{kNm}^2$

$x_1 = \frac{2}{3} l = \frac{2}{3} \times 4 = \boxed{\frac{8}{3}}$

$x_2 = \frac{3}{4} l = \frac{3}{\cancel{4}} \times \cancel{4} = \boxed{3\text{m}}$

~~Slope at B, θ_B~~ =

$a_{AB} = a_1 + a_2 = 240 + 213.33$

$a_{AB} = \boxed{453.33}$

$a_{AB} = (a_1 x_1 + a_2 x_2) = \left(240 \times \frac{8}{3}\right) + (213.33 \times 3)$

$a_{AB}\bar{x} = 1279.9\ \text{kNm}^2$ (or) $1280\ \text{kNm}^2$

Slope at B, $\theta_B = \frac{a_{AB}}{EI} = \frac{453.33}{(2 \times 10^5 \times 8 \times 10^7)}$

$= \frac{453.33}{(16 \times 10^4)}$

$= \cancel{2.833}\ 28.33 \times 10^{-3}$ radians.

Deflection $y_B = \frac{a_{AB}\bar{x}}{EI}$

$= \frac{(1280)}{(1.6 \times 10^4)} = 80 \times 10^{-3}\ \text{m}$

$= \boxed{0.08\ \text{m}}$

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8) A rectangular beam of size 0.3m x 0.4m and 6m span is simply supported at its end. it carry a central point load of 20kN. if young's modulus of materials is 150 kN/mm². calculate the maximum slope maximum deflection.

Given data:

$$= \frac{150 \times 10^3}{}$$

Rectangular beam

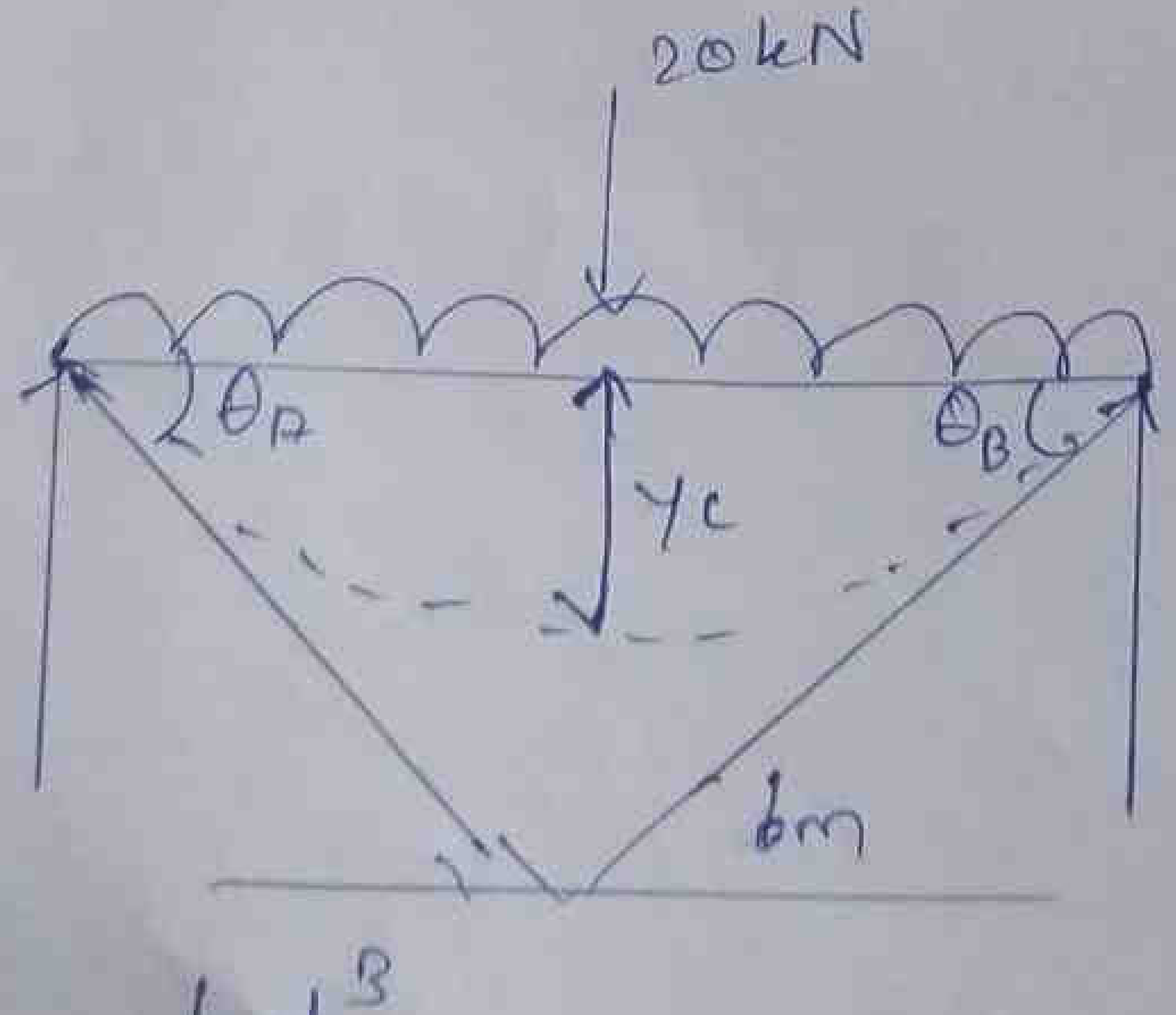
size of beam = 0.3m x 0.4m.

E = 150 kN/mm²

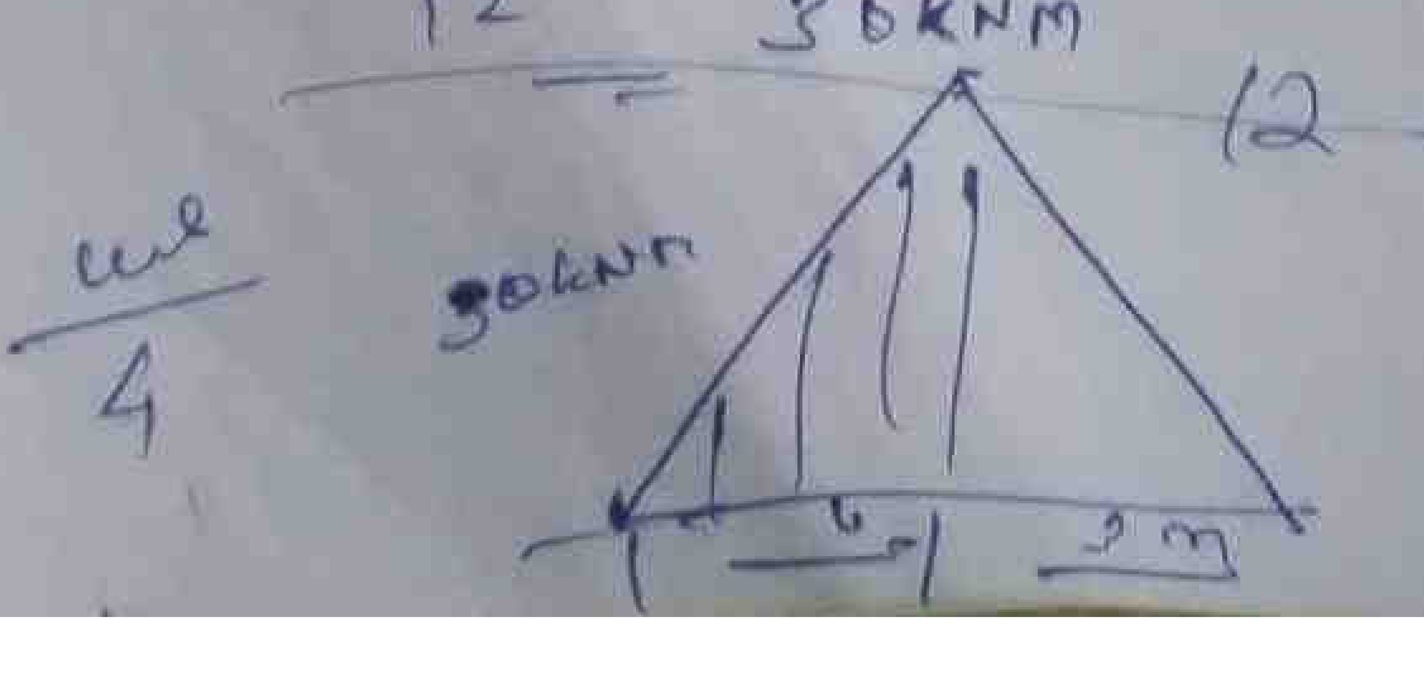
To Find:

Calculate the maximum slope and maximum deflection.

Diagram:



$$I = \frac{bd^3}{12} = \frac{0.3 \times 0.4^3}{12} = 1.6 \times 10^{-3} m^4$$



BMD OF

~~BMD OF~~ $= \frac{wl}{4}$

$= \frac{20 \times 6}{4} = \boxed{30\, kN\, m^2}$

~~Area~~

$a_{AC} = \frac{1}{2} bh$

$= \frac{1}{2} \times 3 \times 30$

$= \boxed{45\, kN m^2}$

$x = \frac{2}{3} \times l$

$= \frac{2}{3} \times 6$

$= \frac{1}{3} \times l$

$= \frac{1}{3} \times 6$

$\boxed{x = 2m}$

$EI = (150 \times 10^3 \times 1.6 \times 10^{-3})$

$\boxed{= 2.4 \times 10^9}$

slope at A $\theta_A = \frac{a_{AC}}{EI}$

$= \frac{\cancel{45}}{(150 \times 10^3 \times 1.6 \times 10^{-3})}$

$= \frac{45}{(2.4 \times 10^9)} \times 2$

$= \boxed{1.875 \times 10^{-8} \text{ radians}}$

Result:

slope at θ_A $= \boxed{1.875 \times 10^{-8} \text{ radians}}$

deflection at $y_C = \boxed{3.75 \times 10^{-8}\, m}$

$$x_c = \frac{2}{3} \times \frac{l}{2} = \frac{l}{3}$$

$$= \frac{6}{3}$$

$$= \boxed{2}$$

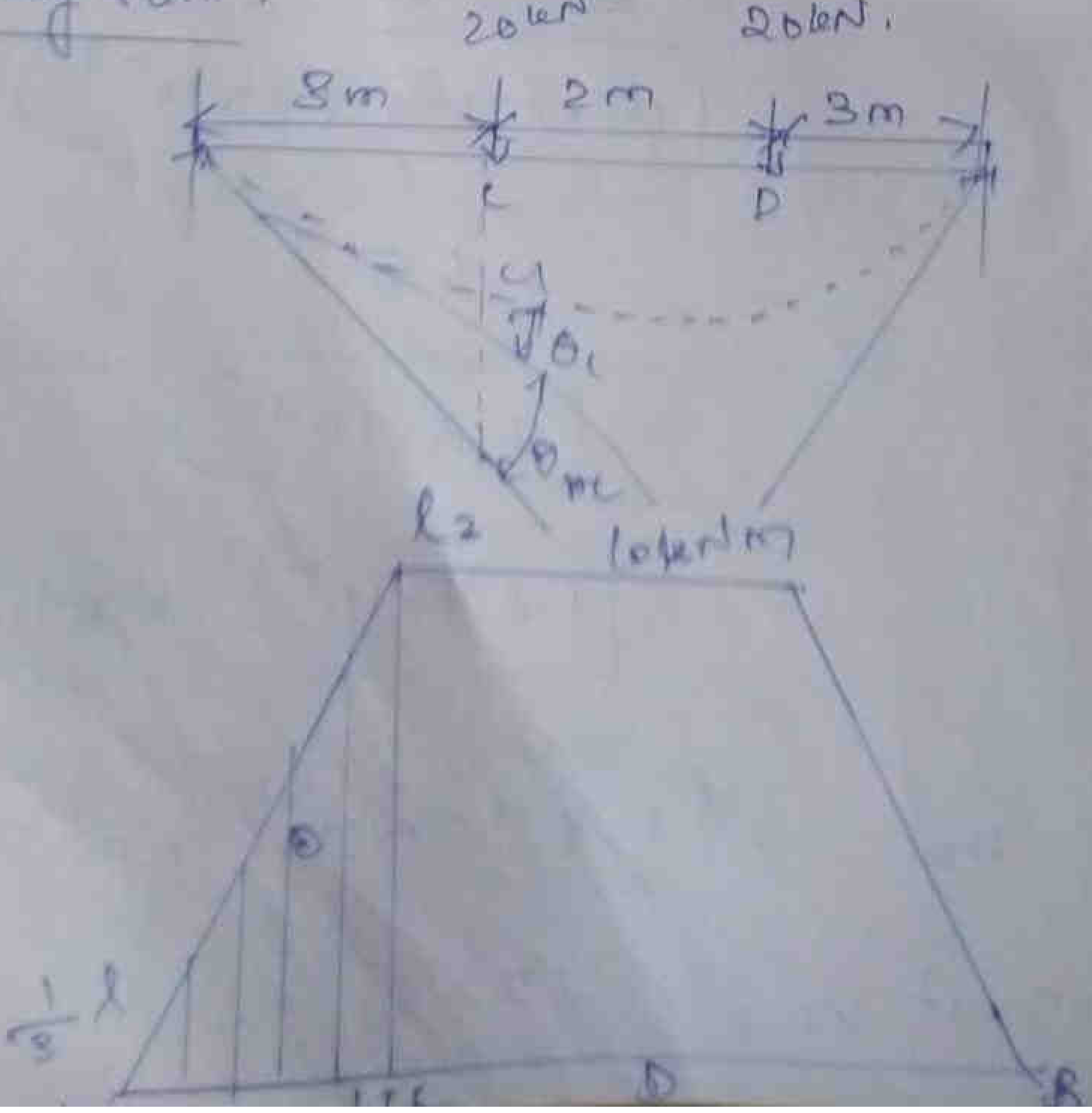
$$y_c = \frac{45}{24 \times 109} \times 2$$

$$= \boxed{3.74 \times 10^{-3}}$$

26.3.22

9) For the beam and bending moment diagram shown in figure. Calculate the slope and deflection at C, if $\theta_A = 7.5 \times 10^{-4}$ radians. and $EI = 2 \times 10^5$ kNm^2.

Diagram:



~~BMD~~

BM

Given data

$\theta_A = 7.5 \times 10^{-4}$ radians

$EI = 2 \times 10^5$ kNm^2

Sol:

$a_A = \frac{1}{2} bh$

$= \frac{1}{2} 3 \times 60$

$= \boxed{90 \text{ kNm}^2}$

slope at $\theta_c = \frac{a_{AC}}{EI}$

$= \frac{90}{2 \times 10^5} = \boxed{4.5 \times 10^{-4} \text{ radians}}$

$a_{AC} = \theta_{AC} - \theta_c$ $\boxed{7.5 \times 10^{-4}}$

~~4.5x10^-4~~ = 7.5 ~~x10^-4~~ -

$\theta_A - \theta_C = \theta_{AC}$

$\theta_C = \theta_{AC} - \theta_A$

$\theta_C = 4.5 \times 10^{-4} - 7.5 \times 10^{-4}$

$\theta_C = 3 \times 10^{-4} = \theta_C.$

$\theta_C = \boxed{3 \times 10^{-4} \text{ radians.}}$

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deflection at $\theta_C = C_1C_2$
$y_{CA} =$

$$= \frac{a_{AC}}{EI} \times x_c$$

$$= \frac{90}{2\times10^5} \times \left(\frac{1}{\cancel{8}} \times \cancel{8}\right)$$

$$= \cancel{3\times10^{-4}}$$

$$= \boxed{4.5\times10^{-4}}$$

From triangle ACC_2

$[\tan\theta = \theta_A]$.

$CC_2 = AC \tan\theta_A$.

$= 3 \times \cancel{\tan}\ 7.5\times10^{-4}$

$= [\ \cancel{\tan\theta} =$

$= \boxed{2.25\times10^{-3} m.}$

Deflection at $C = CC_1 = CC_2 - C_1C_2$

$$= (2.25\times10^{-3} - 4.5\times10^{-4})$$

$$= 1.80\times10^{-3} m$$

$$= \boxed{1.8 mm}$$

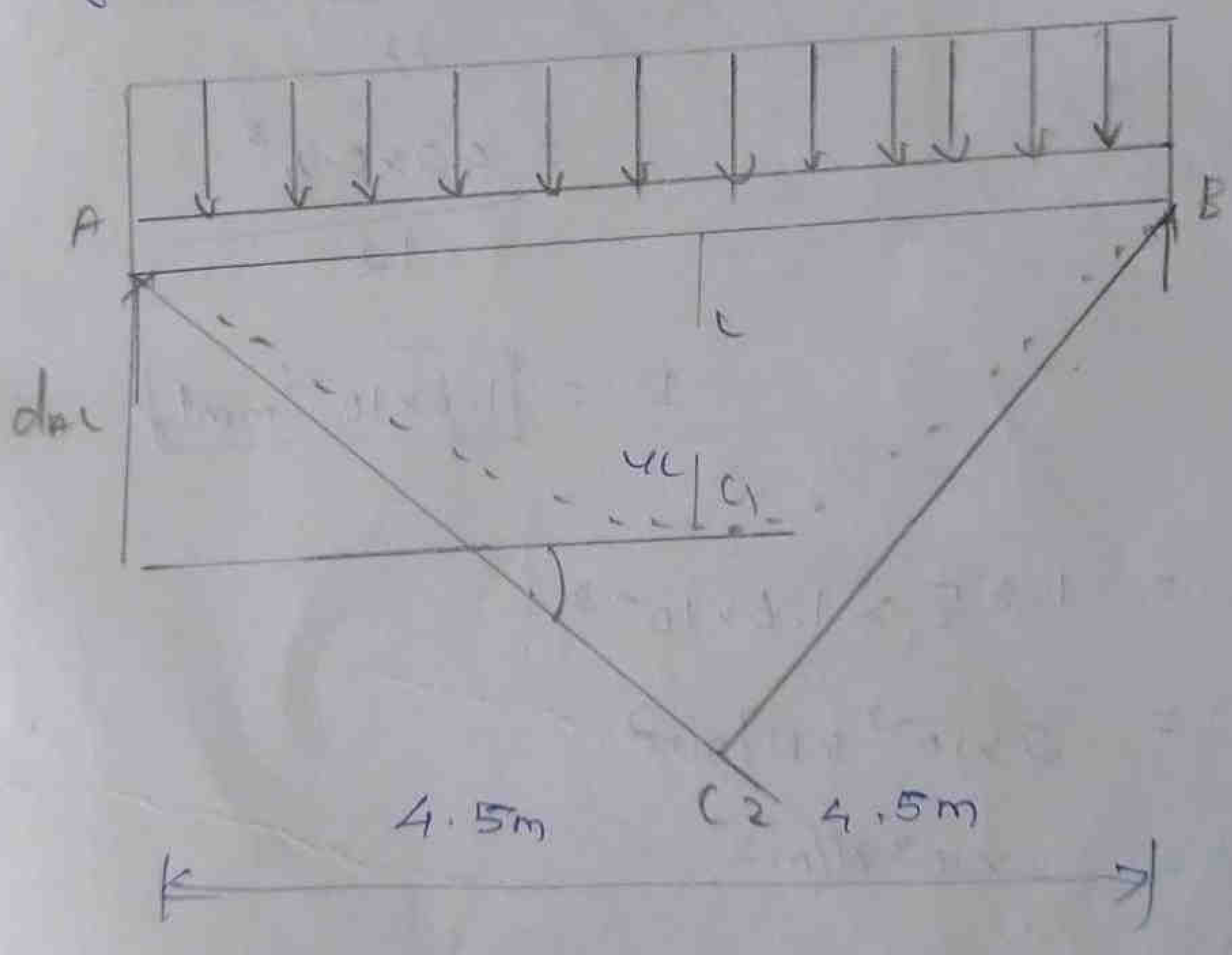
Slope ...

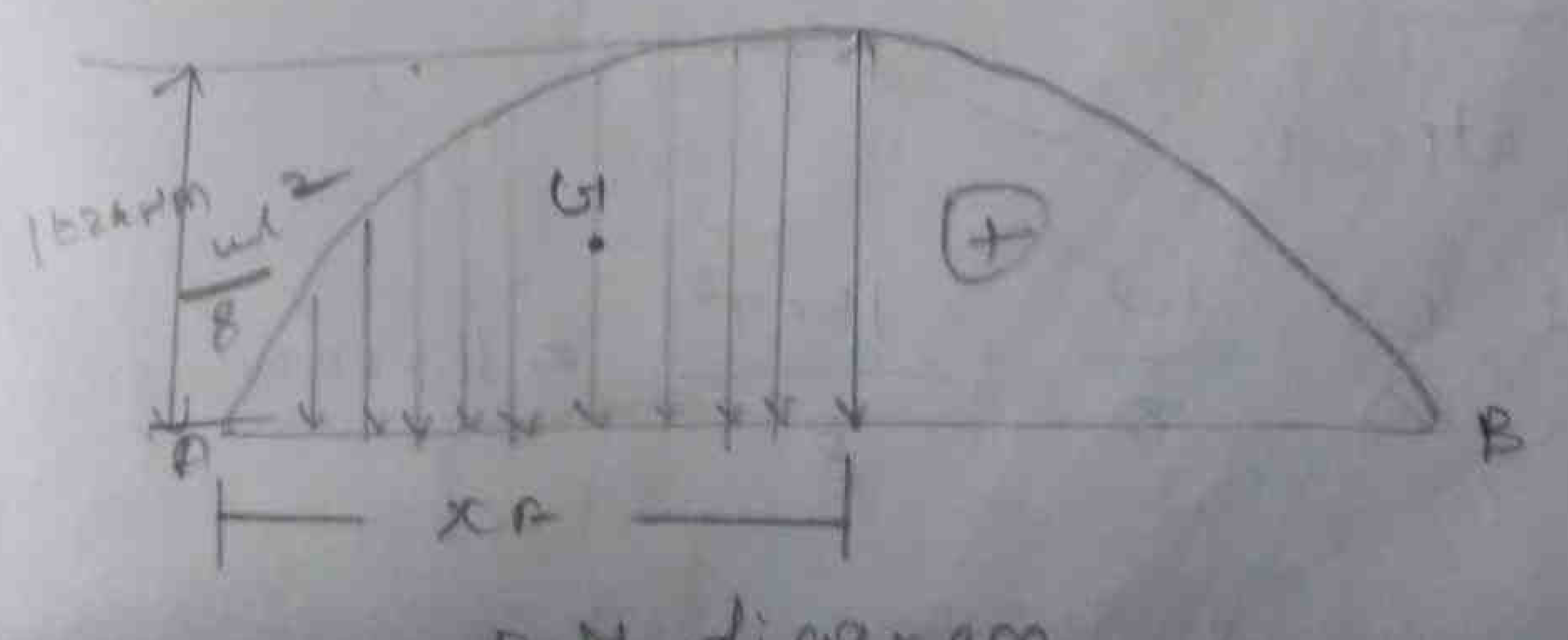
deflection, $y = \boxed{1.8mm}$

propped cantilever Beam

3.22 A simply supported beam of span 9m its 0.3 x 0.4m in cross section it carry is a udl of 16kN to true of the span. If E = young's modulus is = 1.25 kN/mm^2, calculate the maximum slope at the support and maximum central deflection.

Diagram:





B.M diagram.

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c/s size = bxd

= 0.2m x 0.4 m.

udl w = 16kN/m

young's modulus, E = 1.25 kN/mm²

Tofind:

Calculate the slope and deflection.

Solution:

moment of Interia, $I = \frac{bd^3}{12}$

$= \frac{0.2 \times 0.4^3}{12}$

$I = \boxed{1.6 \times 10^{-3} mm^4}$

$EI = 1.25 \times 1.6 \times 10^{-3}$

$= 2 \times 10^{-3}$ kN/mm²

$= 2 \times 10^{3}$ N/m².

BMD

BM at A = 0.

BM at C = $\frac{wl^2}{8} = \frac{16 \times 9^2}{8}$ = 162 kNm.

Given data:

C/S size = b×d

= 0.3m × 0.4 m.

udl w = 16 kN/m

young's modulus, E = 1.25 kN/mm²

To Find:

Calculate the m. slope and m deflection.

Solution:

moment of Interia, $I = \frac{bd^3}{12}$

$= \frac{0.3 \times 0.4^3}{12}$

$I = \boxed{1.6 \times 10^{-3}\ mm^4}$

$EI = 1.25 \times 1.6 \times 10^{-3}$

$= 2 \times 10^{-3}$ kN/mm²

$= 2 \times 10^3$ N/m².

BMD

BM at A = 0.

BM at $C = \frac{wl^2}{8} = \frac{16 \times 9^2}{8} = 162$ kNm.

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Area of BMD at $a_{AC} = \frac{2}{3} bh$

$= \frac{2}{3} \times 4.5m \times 162$ kNm

$= 486$ kN/m^2

slope at $AC = \theta_{AC} = \frac{a_c}{EI} = \frac{a_{AC}}{EI}$

$\frac{486}{2 \times 10^3} = \cancel{242 \times 10}$

$= 0.243$ radians.

Deflection at C, $y_c = \frac{a_{AC}}{EI} x_c$

$x_c = \frac{5}{8} l$

$= \frac{5}{8} \times 4.5$

$= 2.812$ m

$y_c = \frac{486}{2 \times 10^3} \times 2.812$

$= \boxed{683.31 \times 10^{-3} m}$

Result:

slope at C, $\theta_c = 0.243$ radians,

Deflection at $y_c = 683.31 \times 10^{-3}$ m

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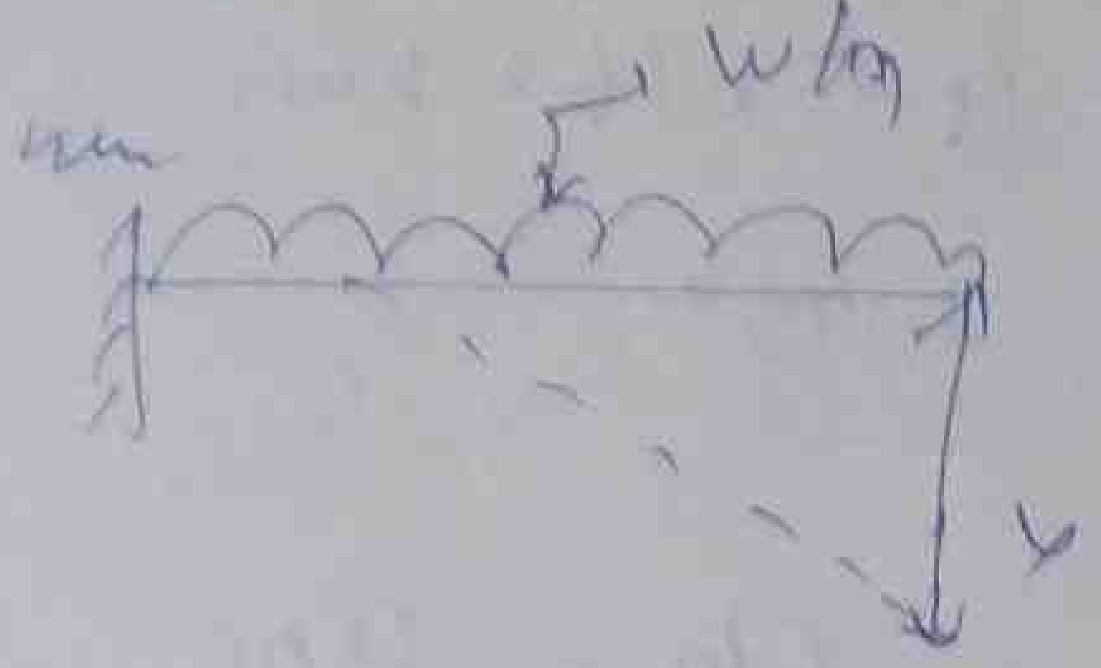
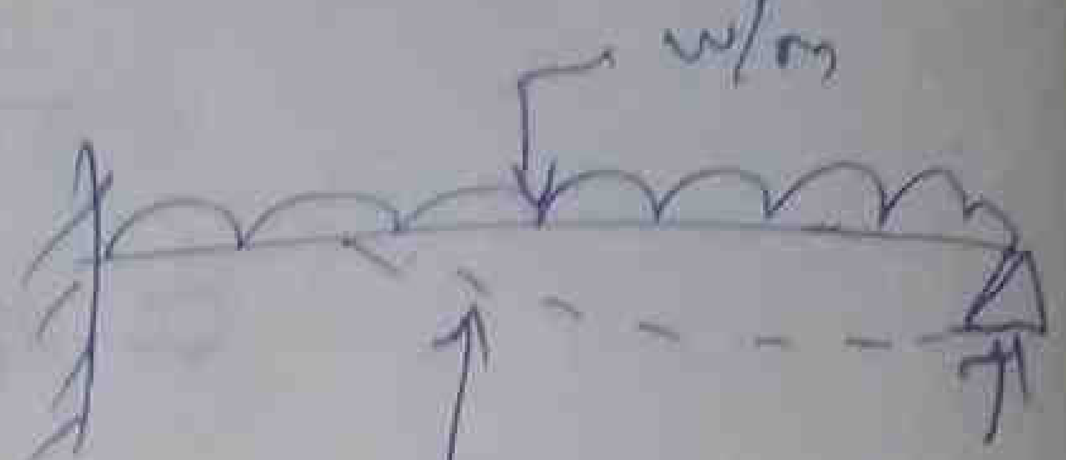
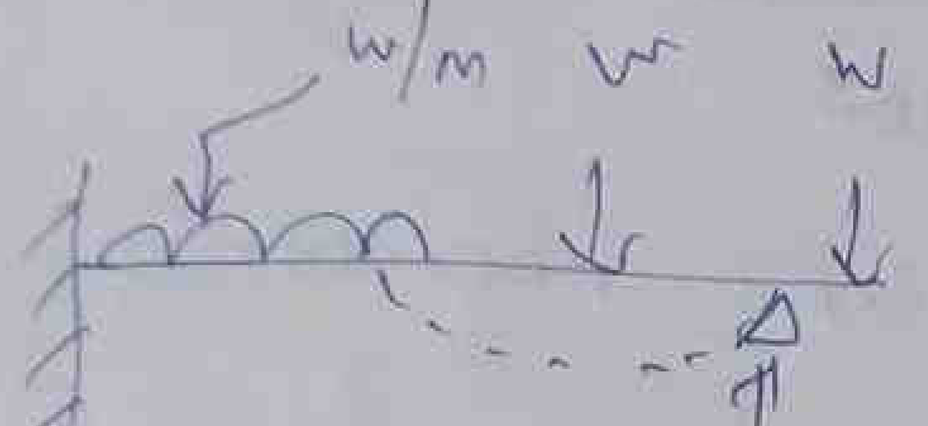
1.2 propped cantilever beam



(1) cantilever beam



propped Elastic Curve.



prop

(2) propped cantilever beam.

(c) Rigid perid (Deflection at prop is zero).

(d) sinking prop (part deflection allowed)

(e) elastic prop.

Types of prop

1. Rigid prop.
2. sinking prop.
3. Elastic prop.

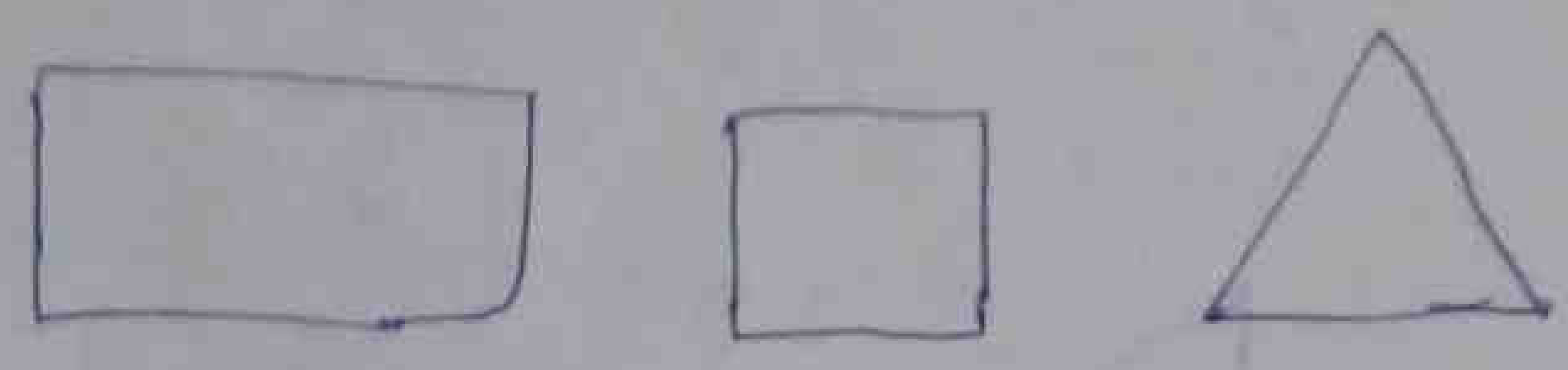
Statically determinate structure and indeterminate structures.

static

$\Sigma V = 0.$

$\Sigma H = 0.$

$\Sigma M = 0.$



Statically determinate structures:

unknown components are solved easily.

$\Sigma V = 0, \quad \Sigma H = 0, \quad \Sigma M = 0.$

(1) cantilever beam, (2) simply supported beam (3) over hanging beam.

Statically indeterminate structure

unknown componentes are not solved easily.

$\Sigma H \neq 0, \quad \Sigma V \neq 0, \quad \Sigma H \neq 0.$

(1) propped cantilever beam.

(2) continuous beam

(3) Fixed beam.

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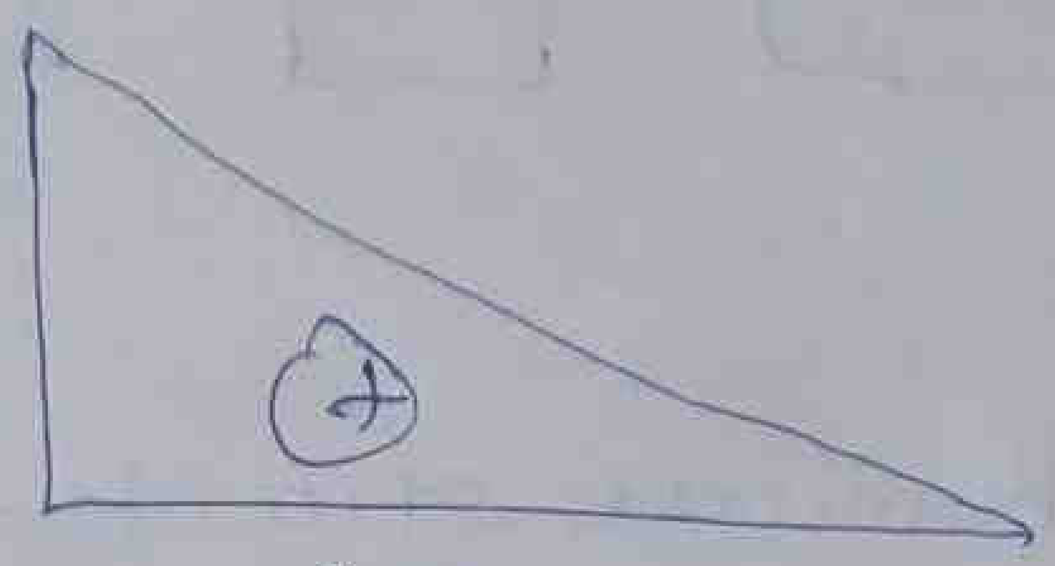
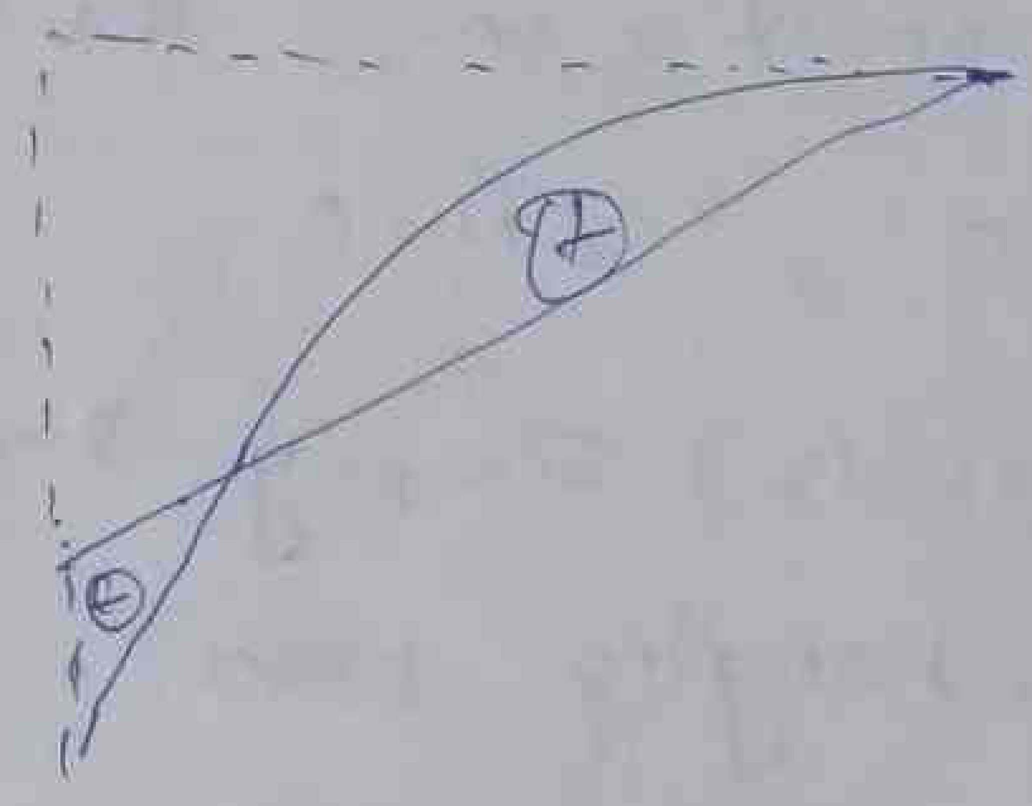
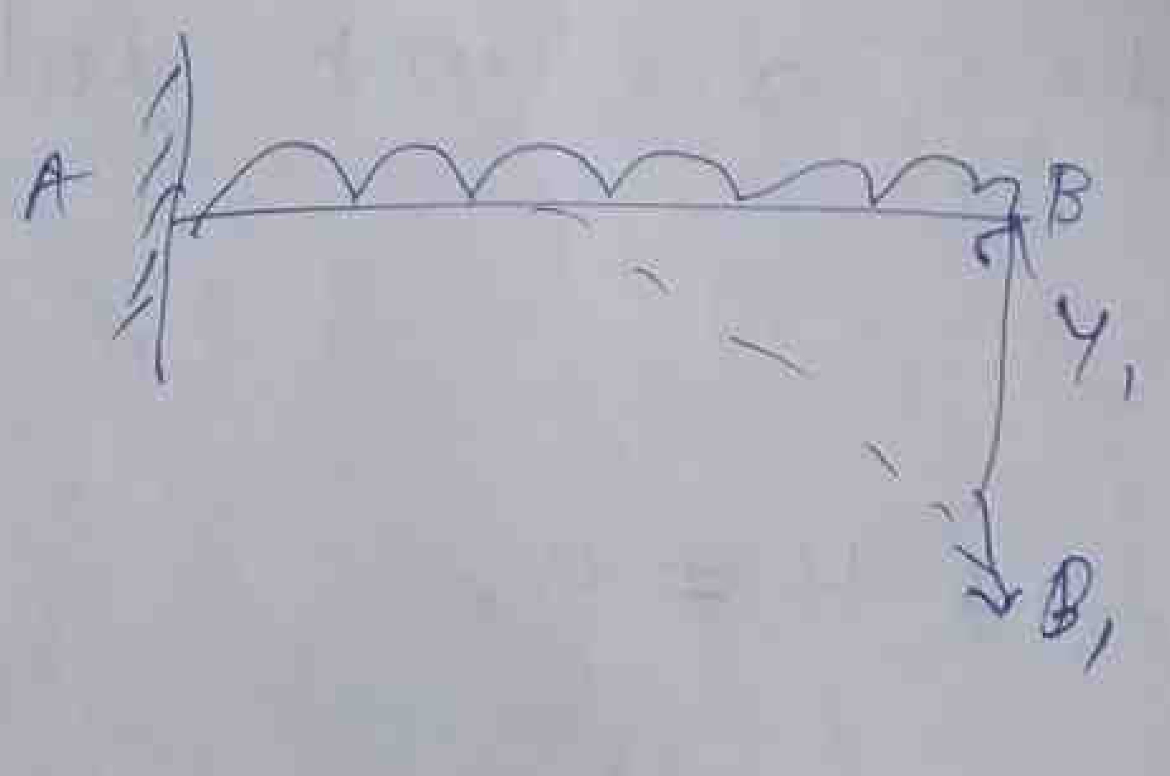
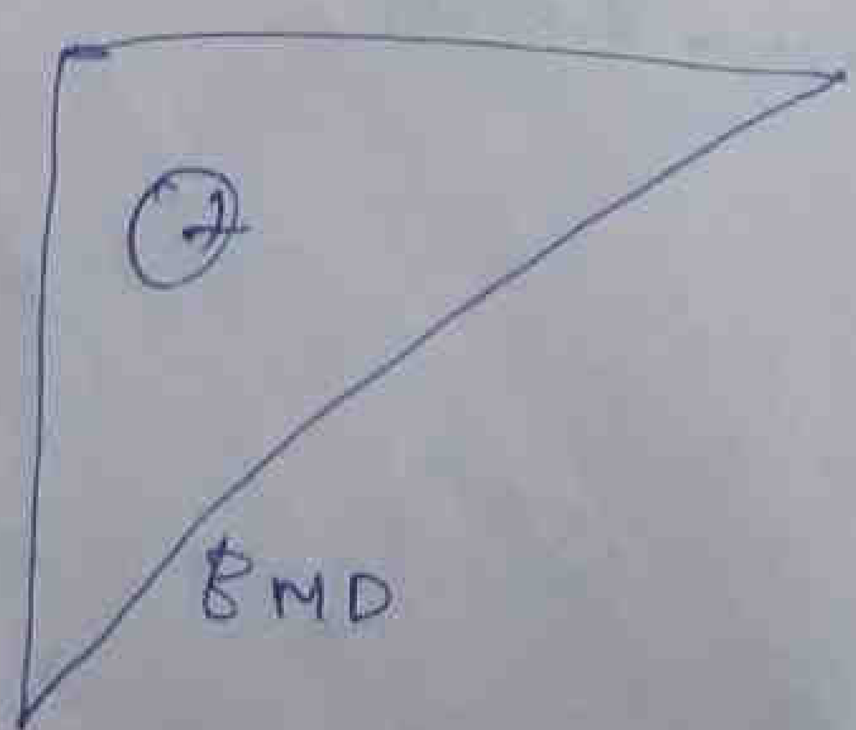


BMD.



propped Cantileveel beam





Cantileeel beam.

$$= \frac{1}{EI} = \left(\frac{1}{2} \times \frac{l}{2} \times \frac{wl}{2}\right)\left(\frac{l}{2} + \frac{2}{3} \times \frac{l}{2}\right)$$

$$= \frac{5wl^3}{48EI}$$

(2) Cantilevel with unknown upward prop reaction V_B at B. as a load.

$$= \frac{a_{AB_2} \; x_{B_2}}{EI}$$

$$= \frac{1}{EI}\left(\frac{1}{2} \times l \times V_{Bl}\right)\left(\frac{2}{3} \times l\right)$$

$$= \frac{V_B l^3}{3EI}.$$

(3) prop reaction, V_B.

Deflection at $B = 0$.

upward deflection = downward deflection.

$$\frac{V_B l^3}{3EI} = \frac{5wl^3}{48EI}$$

$$\boxed{V_B = \frac{5w}{16}}$$

4. Fixed support reaction, V_A.

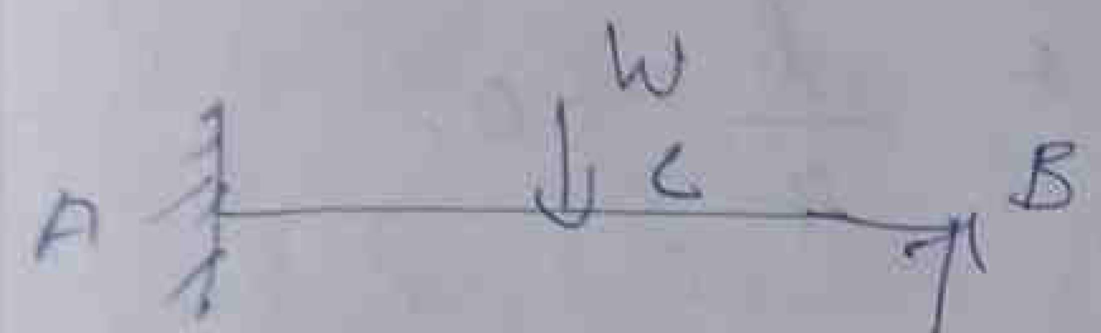
$\Sigma V = 0.$

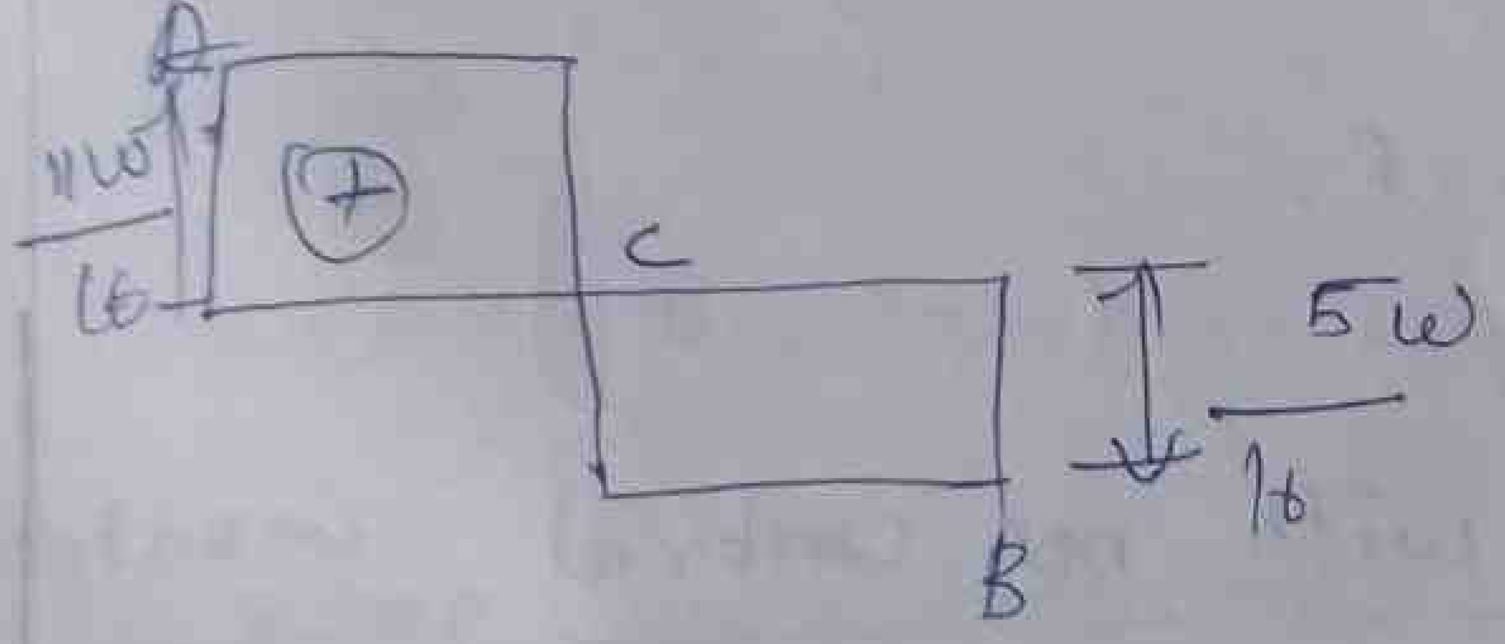
$$V_A = W - \frac{5}{16} w$$

$$= \frac{16w - 5w}{16}$$

$$\boxed{V_A = \frac{11w}{16}.}$$

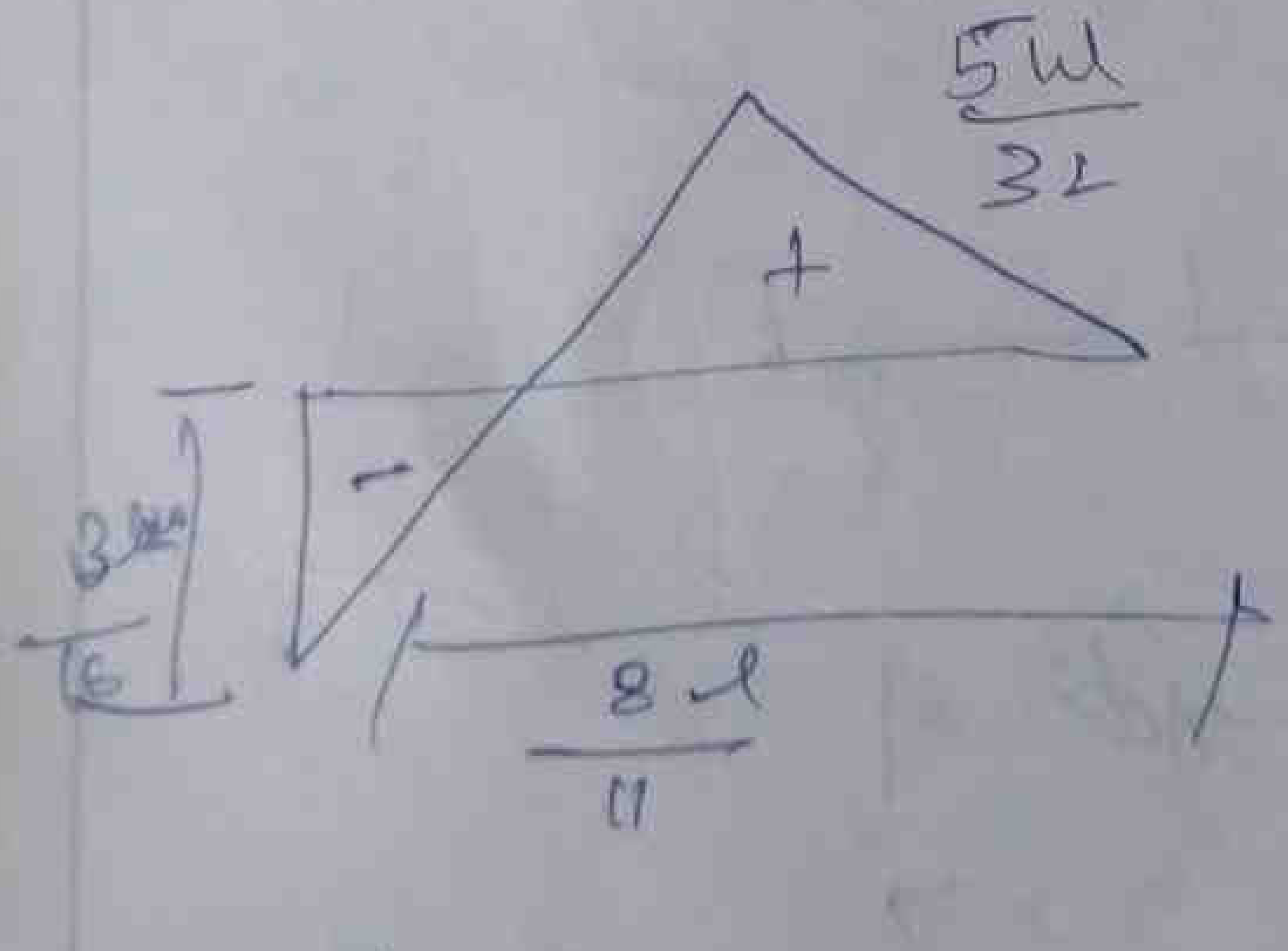
5. Shear force diagram!





$$C = \frac{11w}{16} - w$$

$$= \frac{5w}{16}$$



6. Find BMD

Bm at B = 0

$$C = \left(\frac{5w}{16}\right) \times \frac{l}{2}$$

$$= \frac{5wl}{32}$$

$$BM\ at\ A = \left(\frac{5wl}{16} \times l\right) - \left(w \times \frac{l}{2}\right)$$

$$= \boxed{-\frac{3}{16} wl}$$

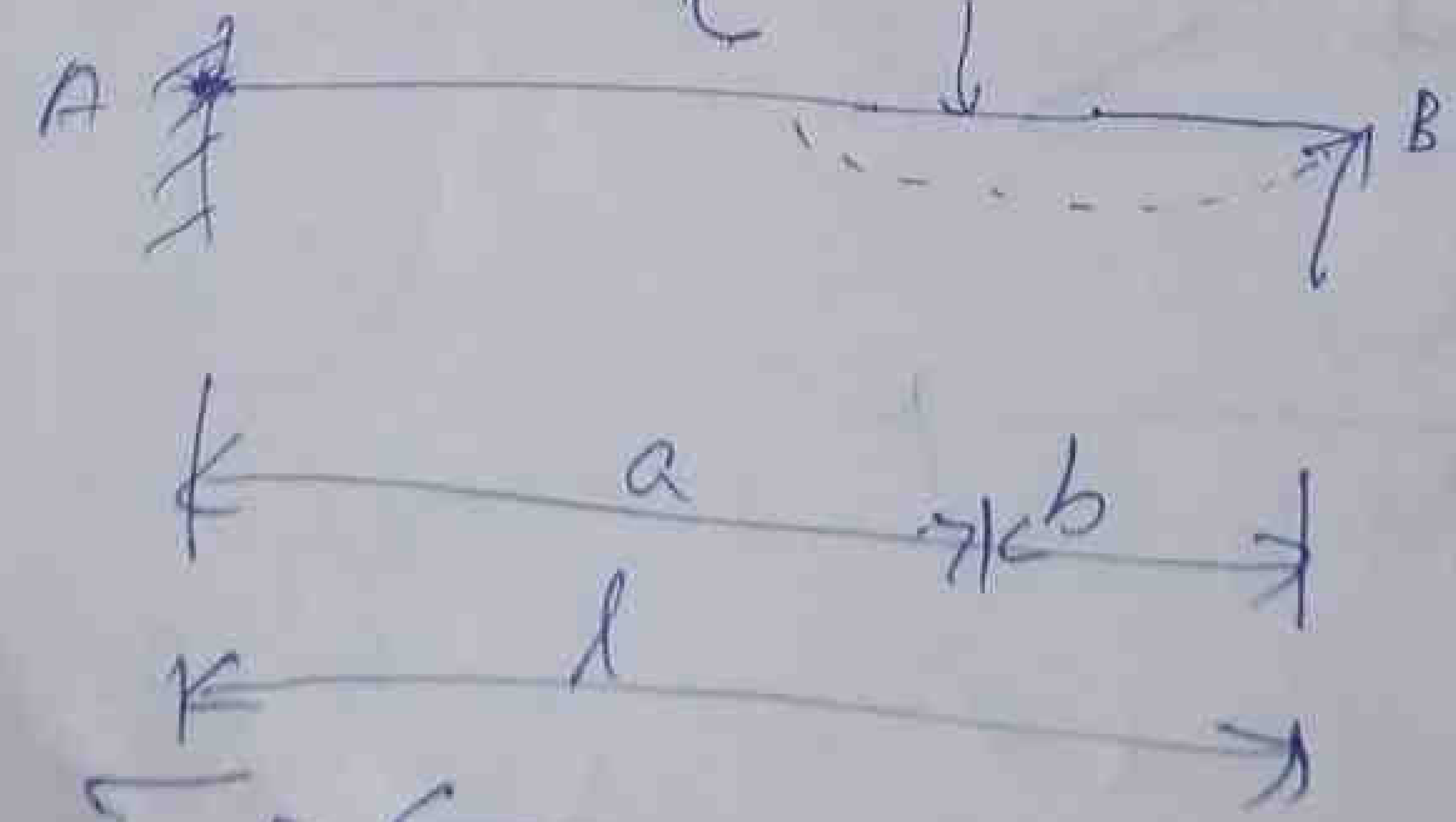
7) point of contra formula.

$$BM\ at\ P = \left(\frac{5}{16} wx\right) - w\left(x - \frac{l}{2}\right) = 0.$$

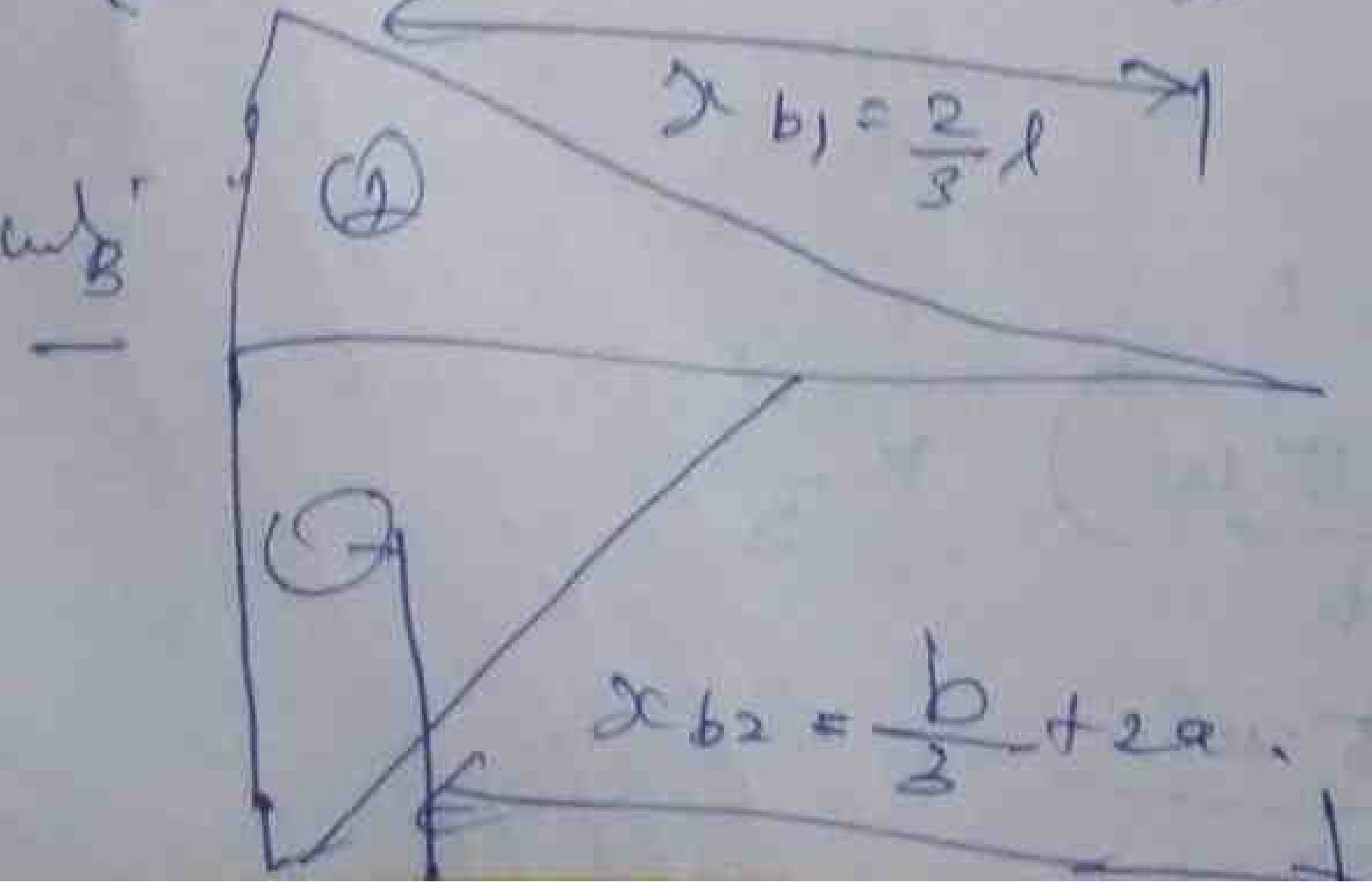
$$(or)\quad x\left(\frac{5}{16} - 1\right) + \frac{l}{2} = 0.$$

$$x = \frac{8}{11} l.$$

propped cantilever with non central concentrated load.



BM by parts



Final BMD

① Bending moment by parts

(i) Due to prop reaction V_B

BM at $B = 0$.

BM at $A = V_B \times l$.

(ii) Due to the external load 'w'

BM at $B = 0$.

BM at $C = 0$.

BM at $A = -Wa$.

② prop reaction V_B.

Tangential Deviation at B.

$$d_{BA} = \frac{a_{AB}\, x_B}{EI} = 0.$$

$$\left(\frac{1}{2} \times l \times V_B\, l\right)\left(-\frac{1}{2}\, a \times wa\right)\left(b + \frac{2a}{3}\right) = 0$$

$$\frac{V_B l^2}{3} = \frac{wa^2}{6}(3b+2a)$$

$$V_B = \frac{wa^2}{2l^3}(2l+b)$$

3) Fixed support reaction, V_A.

$\Sigma V = 0.$

$V_A = W - V_B$

$= W - \frac{Wa^2}{2l^3}(2l+b)$

$= \frac{W}{2l^3}(2l^3 - 2a^2 l - a^2 b).$

$= \frac{W}{2l^3}\left[2l(l-a)(l+a) - a^2 b\right].$

$= \frac{Wb}{2l^3}\left[2l^2 + 2al - a^2\right]$ $\boxed{\because l - a = b}$

$= \frac{Wb}{2l^3}\left[2l^2 + a(2l-a)\right].$

$= \frac{Wb}{2l^3}\left[2l^2 + (l-b)(l+b)\right] = \frac{Wb}{2l^3}\left[3l^2 - b^2\right]$

4) SF diagram

SF at $B = -W_B$

SF at $A = +V_A$

5) Final Bending moment

B.M at $B = 0.$

B.M at $A = V_B \times l - Wa.$

$$= \frac{wa^2}{2l^3}(2l+b) \times l - wa.$$

$$= \frac{wa}{2l^2}[2al + ab - 2l^2)$$

$$= \frac{wa}{2l^2}[ab - 2l(l-a)].$$

$$= \frac{wab}{2l^2}[a - 2l].$$

$$= \frac{wab}{2l^2}[2l - a].$$

$$= \frac{-wab}{2l^2}(a+b).$$

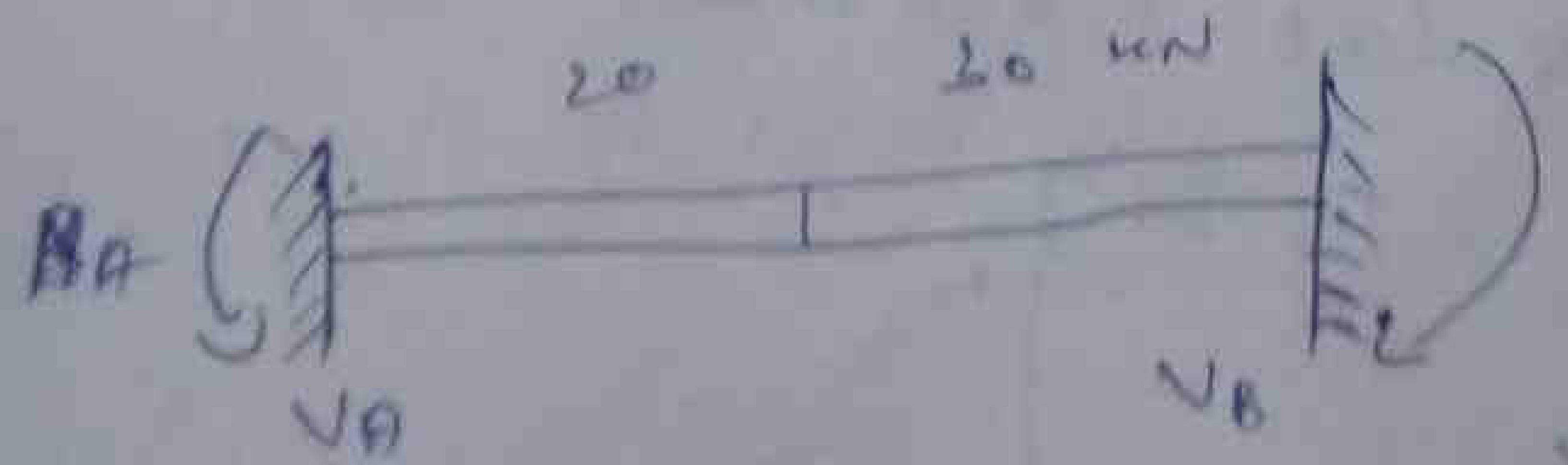
BM at C = $V_B \times V_b$

$$= \frac{wa^2}{2l^3}(2l+b) \times b$$

$$= \frac{wa^2 b}{2l^3}(2l+b).$$

Unit - 2

2.1 Fixed Beam



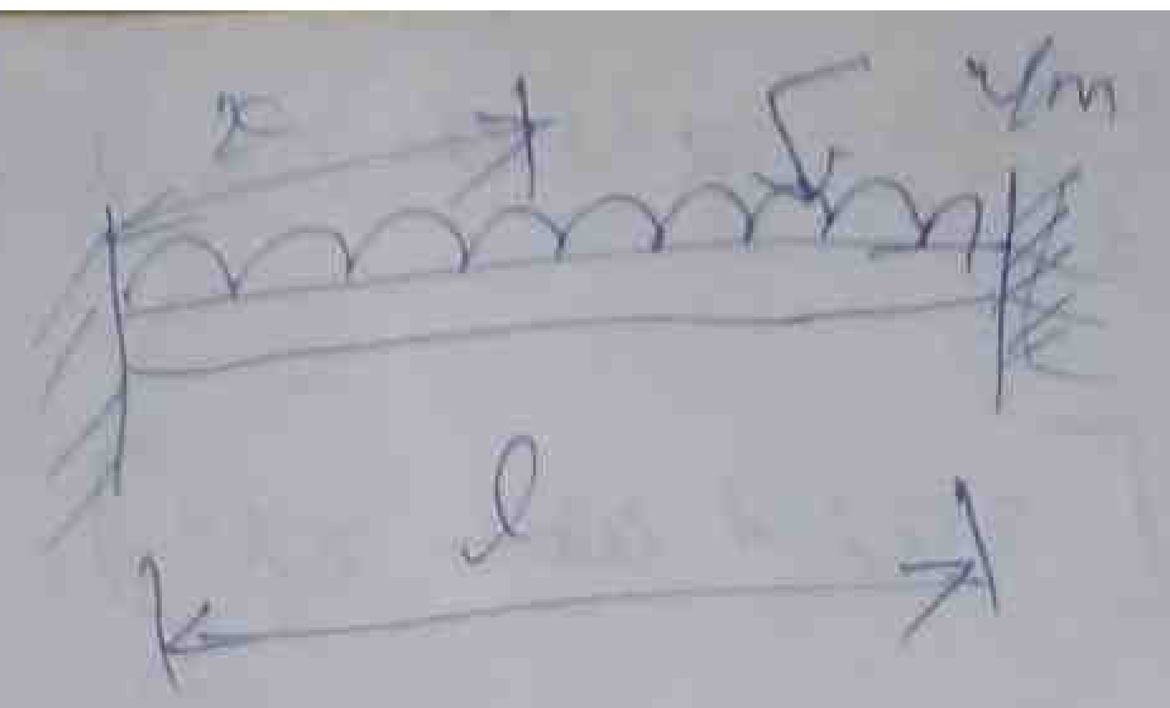
Symmetrical condition of beam.

V = 50 kN

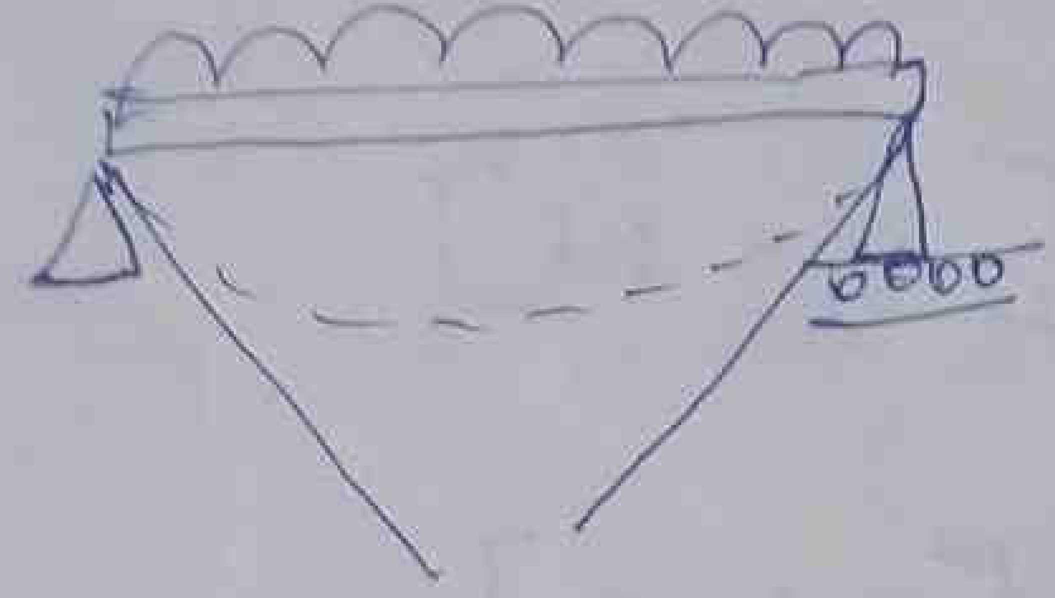
$V_A = 25$ $M_A =$

$V_B = 25$ $M_B =$

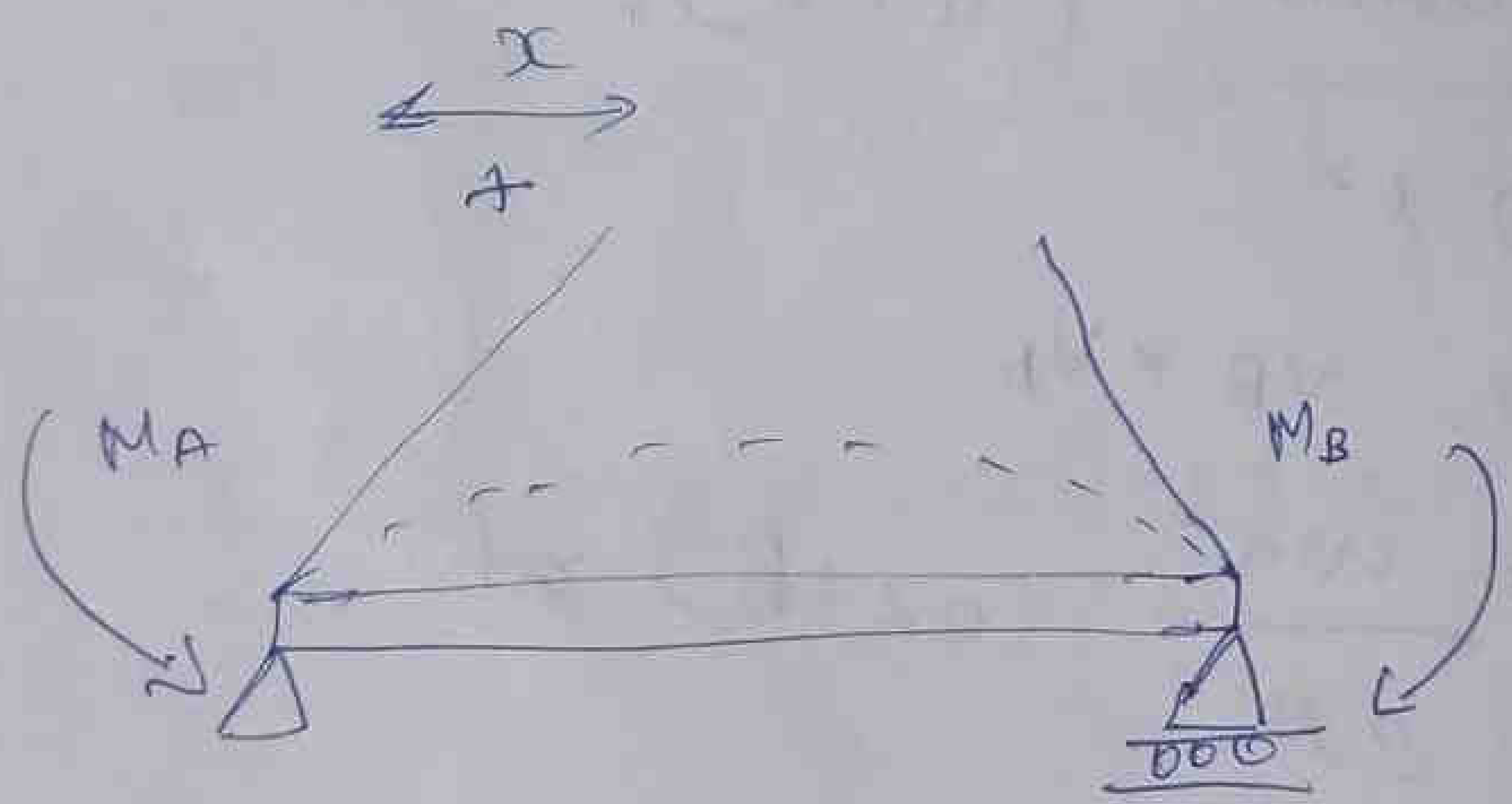
$M = M_A = M_B.$



(a) Fixed beam

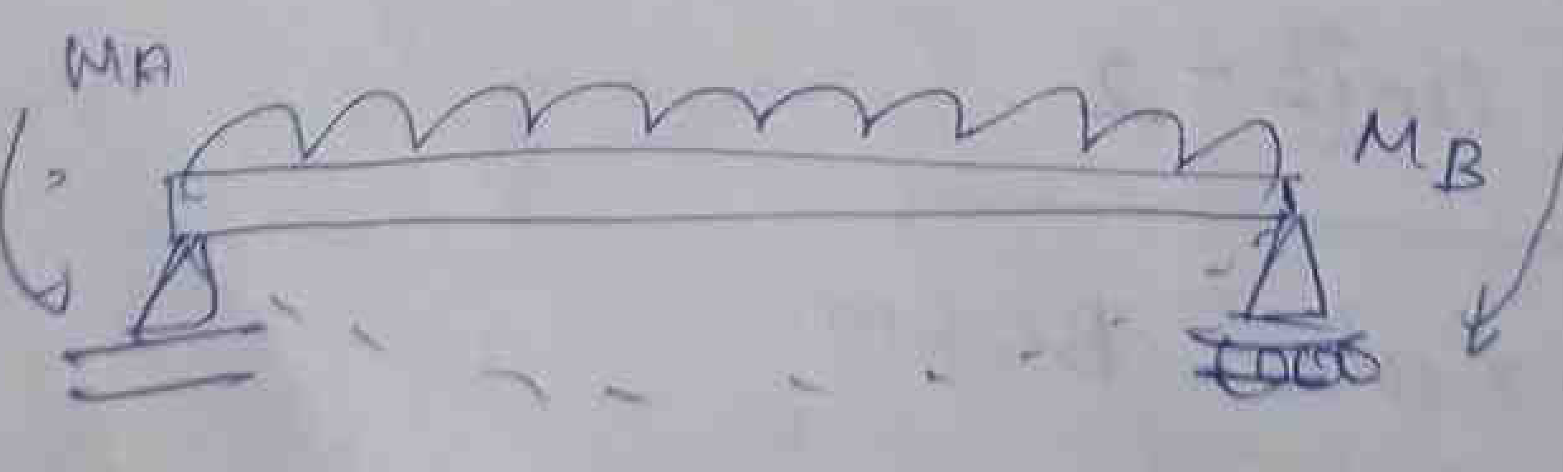


(b) simply supported beam with loading



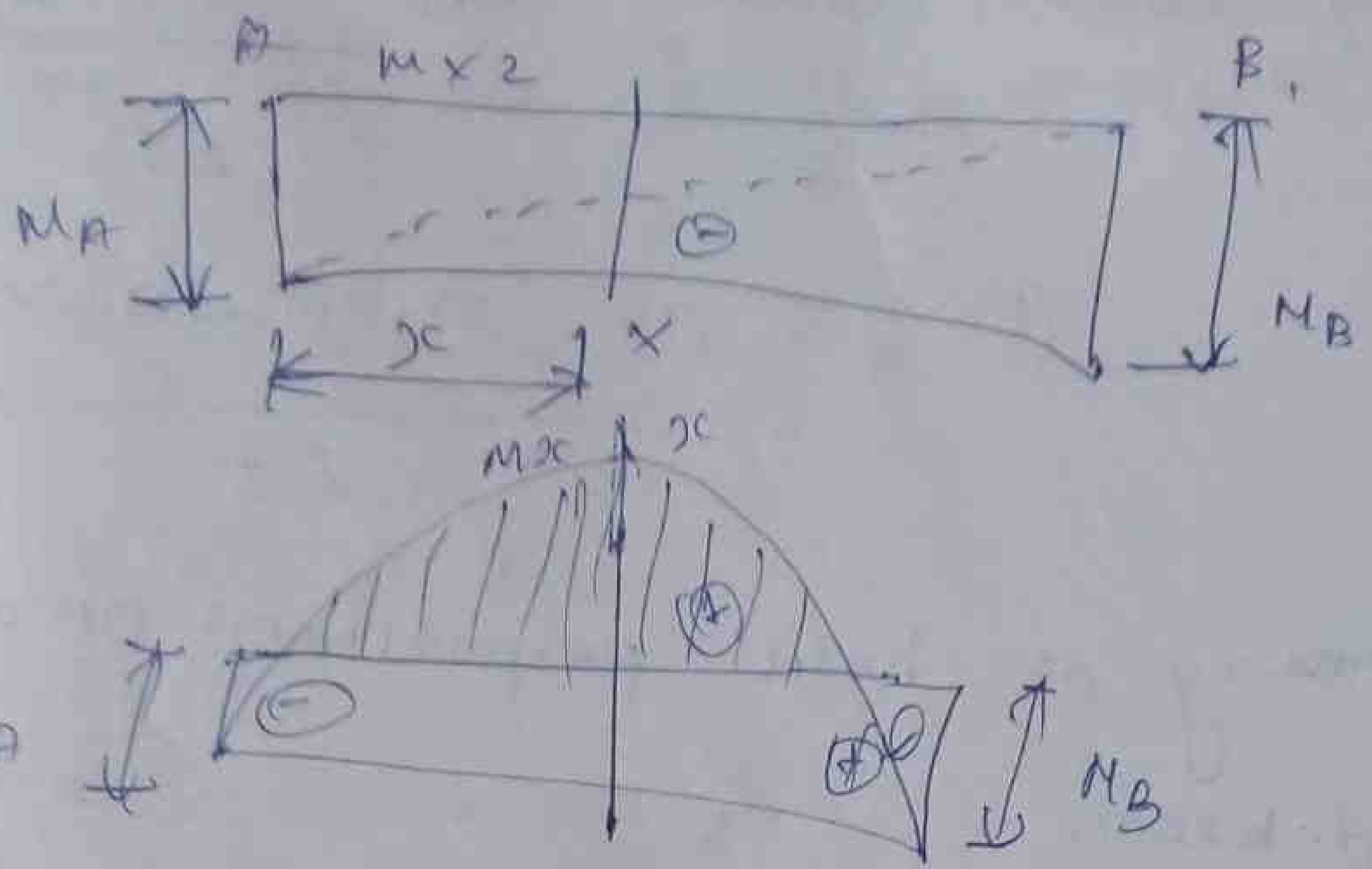
(c) ~~sim~~ S.S beam with end moment

(d)



Equivalent Fixed beam.

BMD



Find BMD

3-22.

1) If the beam is symmetrically load $M_A = M_B$ and $V_A = V_B$.

2) $\theta_B - \theta_A = \theta_{AB}$ = change of slope between A and B = 0.

when $x = 0$, $\frac{dy}{dx}$ = slope at A = $\theta_A = 0$ and

y = Deflection at A = $y_A = 0$.

Then, $a_{AB} = a_1 - a_2$ and $a_{AB}\, x_A = a_1 x_1 - a_2 x_2$

Now apply Mohr's area moment theorems

By First theorem. $\theta_{AB} = \frac{a_{AB}}{EI}$

(or).

$0 = \frac{a_1 - a_2}{EI}$ (or) $a_1 - a_2 = 0$ (or)

$a_1 = a_2$

By Second Theorem . $d_{AB} = \dfrac{a_{AB}\, x_n}{EI}$

(or) $\quad 0 = \dfrac{a_1 x_1 - a_1 x_2}{EI}$

1) Summery of basic propositions of a fixed beam.

2) Moment of Free BMD area = Moment of Fixed BMD area ($a_2 x_2$) about the same support.

3) Further from $a_1 = a_2$ and $a_1 x_1 = a_2 x_2$, $x_1 = x_2$
(i.e centroidal distance = moment distance of fixed of free BMD (x_1) BMD from the same support).

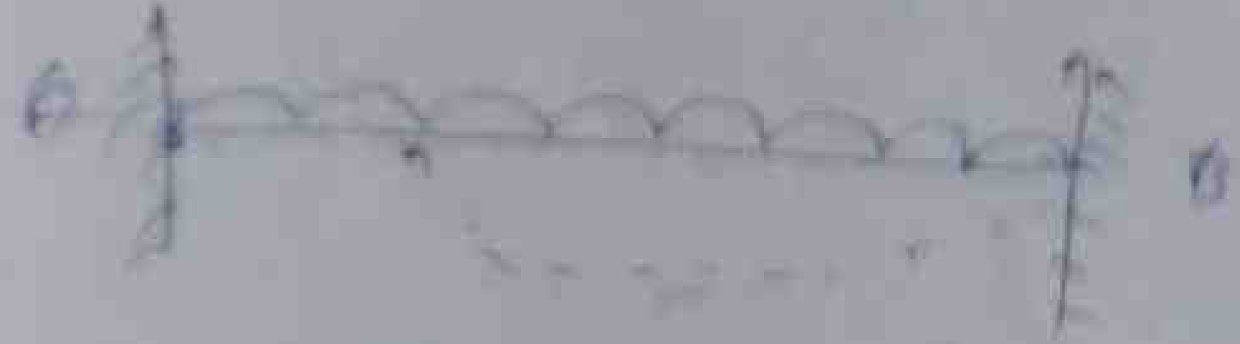
4) $M_A + M_B = \dfrac{2a_1}{l}$

5) $M_A + 2M_B = \dfrac{6a_1 x_1}{x_2}$

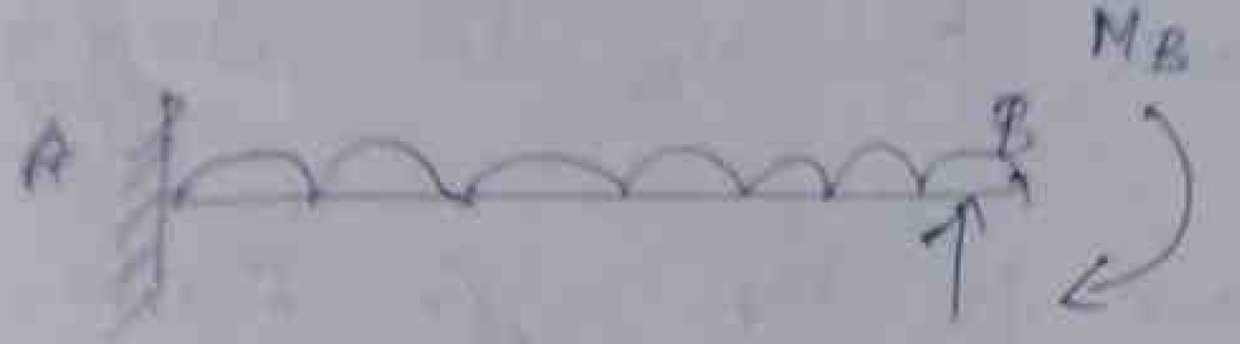
6) From (4) and (5)

$M_A \; \dfrac{2a_1}{l^2}(2l - 3x)$ and $M_B = \dfrac{2a_1}{l^2}(3x - l)$

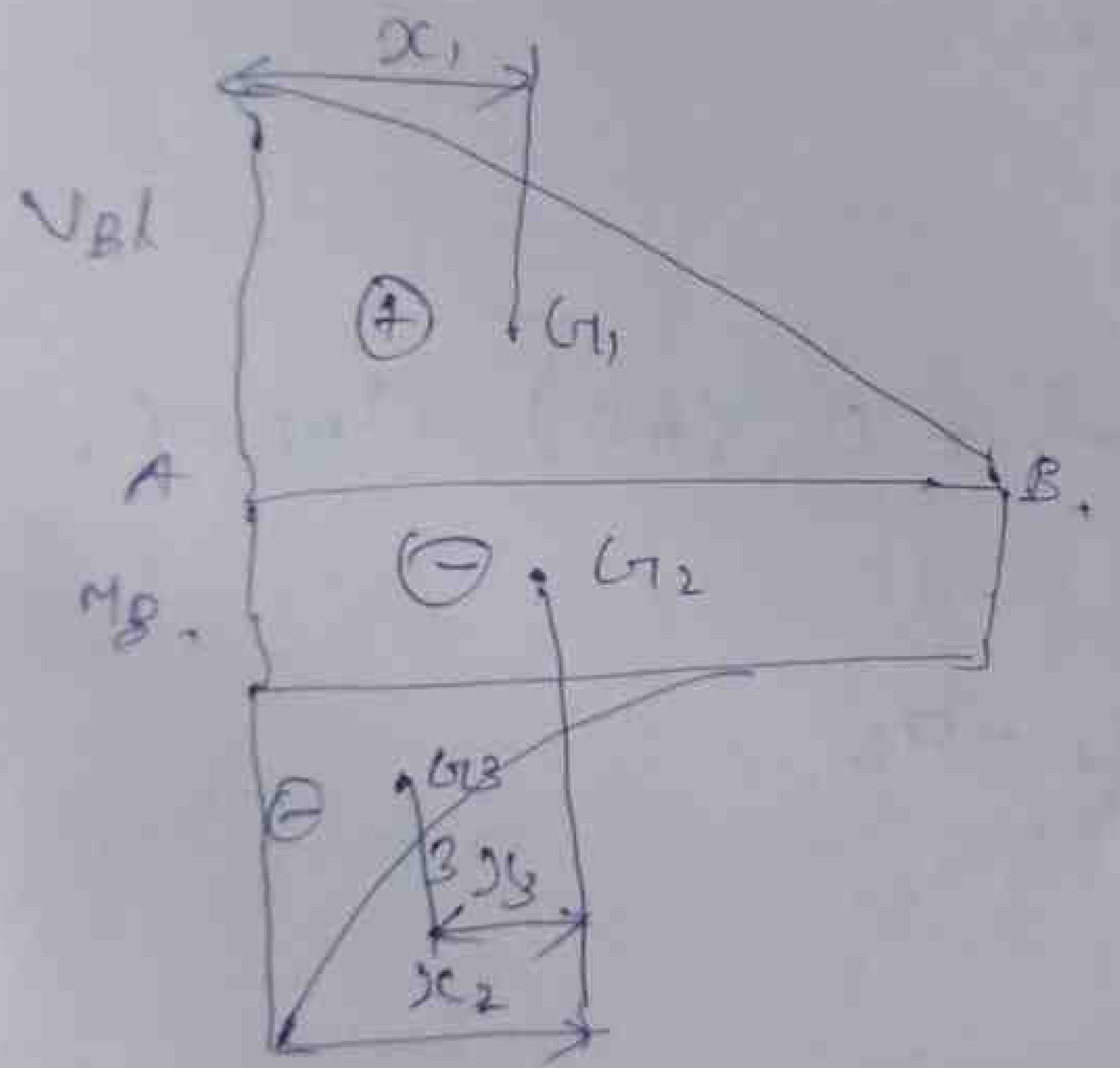
Analysis vertical reaction and fixing moment of any one as redundant.



(a) Fixed beam.



(b) Equivalent cantilever beam.



Consider a loaded fixed beam AB of span l.

Let M_A & M_B be the fixed end moments.

V_A & V_B are the vertical reactions

Slope, $\theta_A = 0 = \theta_B$ and

Deflection, $y_A = 0 = y_B$.

Area and centroid of BMD

a_1, a_2, a_3 be the area of BMD of ①, ② & ③ and $x_1, x_2, \& x_3$ are their centroidal distances from A.

The moment due to V_A is sagging & the moment due to M_A & External load are hogging.

$B = \theta_{AB} = 0$.

$$\theta_{AB} = \frac{a_{AB}}{EI} = 0 \text{ (or) } a_{AB} = 0.$$

$a_{AB} = a_1 - a_2 - a_3$.

proposition: 1

Area of sagging moment Diagrams = Area of hogging moment diagram.

$$d_{AD} = 0 = \frac{a_{AB} x_B}{EI} \text{ (or) } a_{AB} x_B = 0$$

$$a_{AB} x_B = a_1 x_1 - a_2 x_2 - a_3 x_3.$$

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proposition: 2

moment of sagging BMD = moment of hogging BMD.

Analysis of standard cases of fixed beams.

Fixed beam with central concentrated load

span AB = l.

central concentrated load = w.

let

M_A & M_B be the hogging fixed end moments.

V_A & V_B be the vertical support reaction.

consider M_A & M_B as redundant.

1) Equivalent simply supported beam.

Remove the fixed supports.

consider the beam as simply supported with central load 'w' and redundant moments M_A & M_B at A and B.

2) Free BMD.

BM at A = 0.

BM at B = 0.

BM at C = $\frac{wl}{4}$

Area of BMD

$$a_1 = \frac{1}{2} l \times \frac{wl}{4} = \frac{wl^2}{8}.$$

3) Fixed BMD

Consider the simply supported beam with redundant moments M_A and M_B

Due to symmetry

$M_A = M_B = M$

Say

BMD is a rectangle.

1.04. 2022

Area of BMD, $a_2 = M \times l$

4) Fixed end moments

(Redundant moments M_A & M_B)

Tangents through A and B coincide

$\therefore \theta_B = \frac{a_{AB}}{EI} = 0.$

(or) $a_{AB} = 0.$

But, $a_{AB} = a_1 - a_2$

$\therefore a_1 - a_2 = 0.$

(or) $\frac{wl^2}{8} \cdot Ml = 0.$

$\therefore a_1 - a_2 = 0.$

(or) $\frac{wl^2}{8} - Ml = 0.$

(or) $M = \frac{wl^2}{8}$

$\therefore M_A = M_B.$

$= \frac{wl}{8}$ (Hog)

5) <u>vertical support reactions</u>

V_a and V_B.

since loading is symmetrical

$V_A = V_B = \frac{W}{2} \uparrow$

6) <u>SF diagram</u>

SF. diagram is shown in.

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7) Final Bending moments

$$M_A = M_B = \frac{-wl}{8}.$$

$$M_C = \frac{-wl}{8} + \left[V_B \times \frac{l}{2} \right] = -\frac{wl}{8} + \frac{w}{2} \times \frac{l}{2}$$

$$= \frac{-wl}{8}.$$

8) BM at $P_1 = -M_A + V_A x_1 = 0.$

$$\text{i.e.} -\frac{wl}{8} + \frac{w}{2} x_1 = 0 \text{ (or) } x_1 = \frac{l}{4}$$

BM at $P_2 = -M_B + V_B x_2 = 0.$

$$\text{i.e.} -\frac{wl}{8} + \frac{w}{2} x_2^2 = 0 \text{ (or) } x_2 = \frac{l}{4}$$

points of contra flexures are at $1/4$ from either.

9) Deflection at C

Due to symmetry consider half of beam and BMD maximum at C

$$y_C = d_{CB} = \frac{a_{CB}\, x_C}{EI}$$

$$= \frac{1}{EI} \left[\frac{1}{2} \times \frac{l}{2} \times \frac{wl}{4} \right] \cdot \left[\frac{1}{3} \times \frac{l}{2} \right] \left[-\frac{l}{2} \times \frac{wl}{8} \right]$$

$$= \left[\frac{1}{2} \times \frac{l}{2} \right]$$

$$= \frac{1}{EI} \left[\frac{wl^3}{96} - \frac{wl^3}{64} \right].$$

$$= -\frac{wl^3}{192EI}.$$

negative sign indicates deflection is down words.

Note:

For a simply supported beam with central point load.

i) Max (+) ve BM at centre $= \frac{wl}{4}$.

and BM at supports $= 0$.

ii) maximum deflection m_c at centre $= \frac{wl^3}{48EI}$

Hence by making the ends fixed. The beam is twice stronger more length of the beam is detfecuive inrestion.

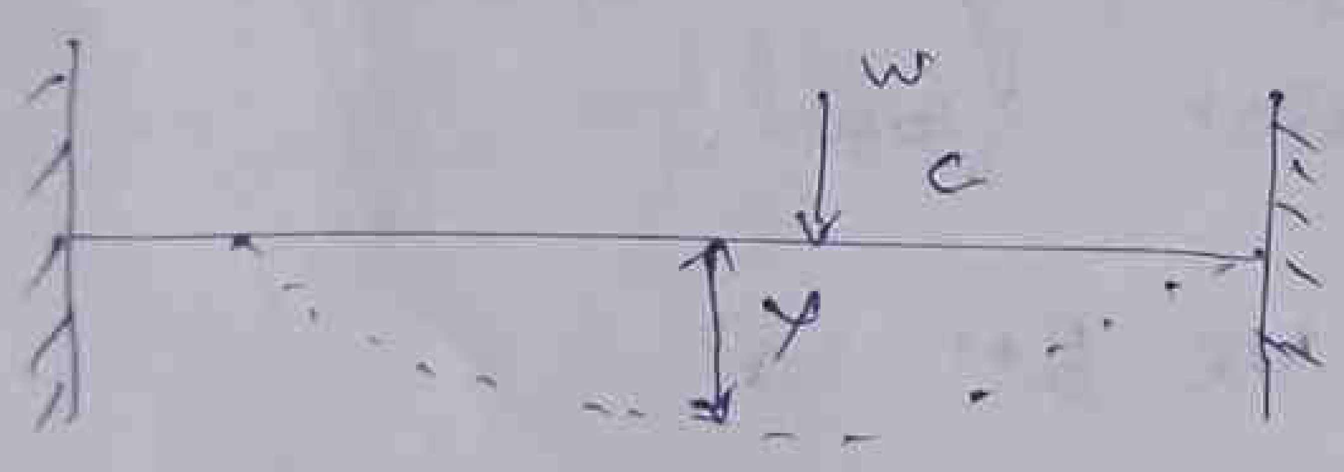
By BM and Fault times stiffness then a simply supported beam subjected to central point load.

Fixed beam with non concentrated load

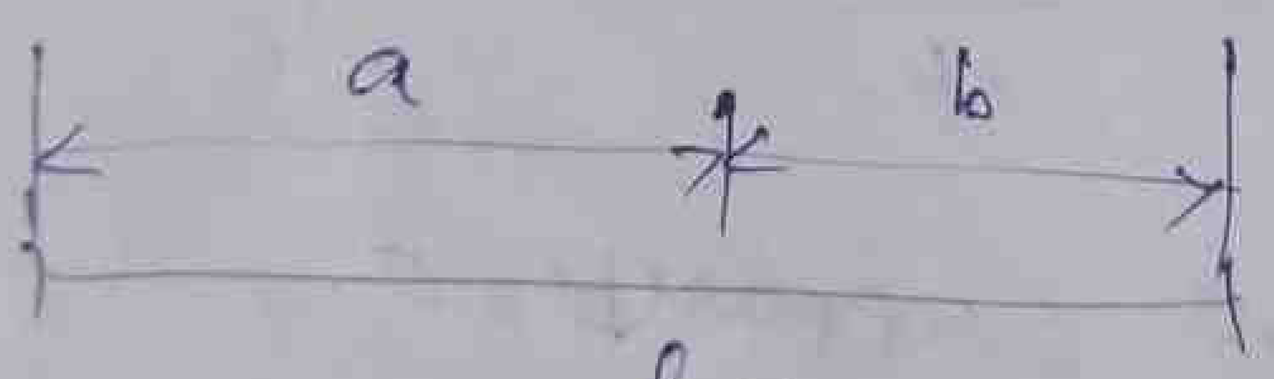
V_A & V_B are vertical reactions.

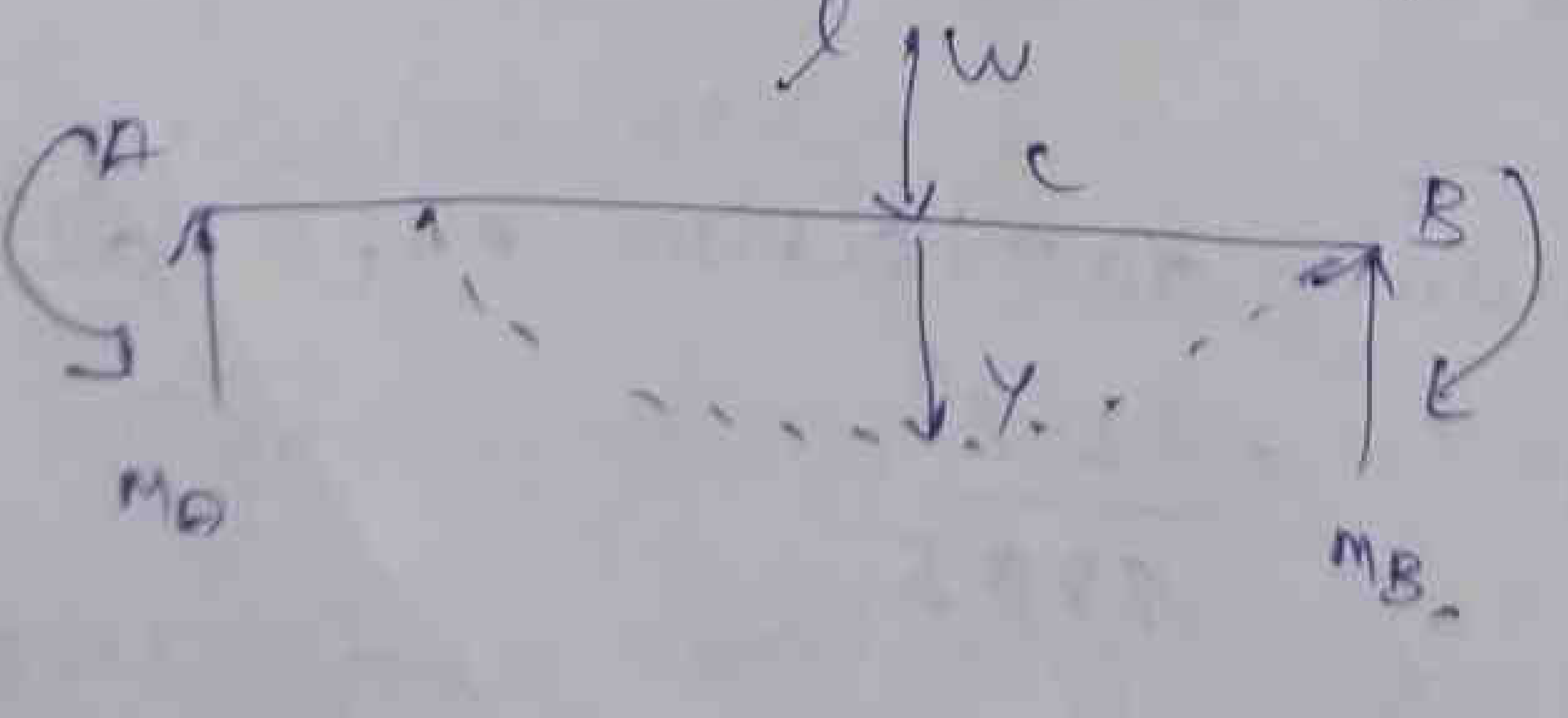
M_A & M_B be hogging fixed moments.

consider M_A & M_B as redundants.

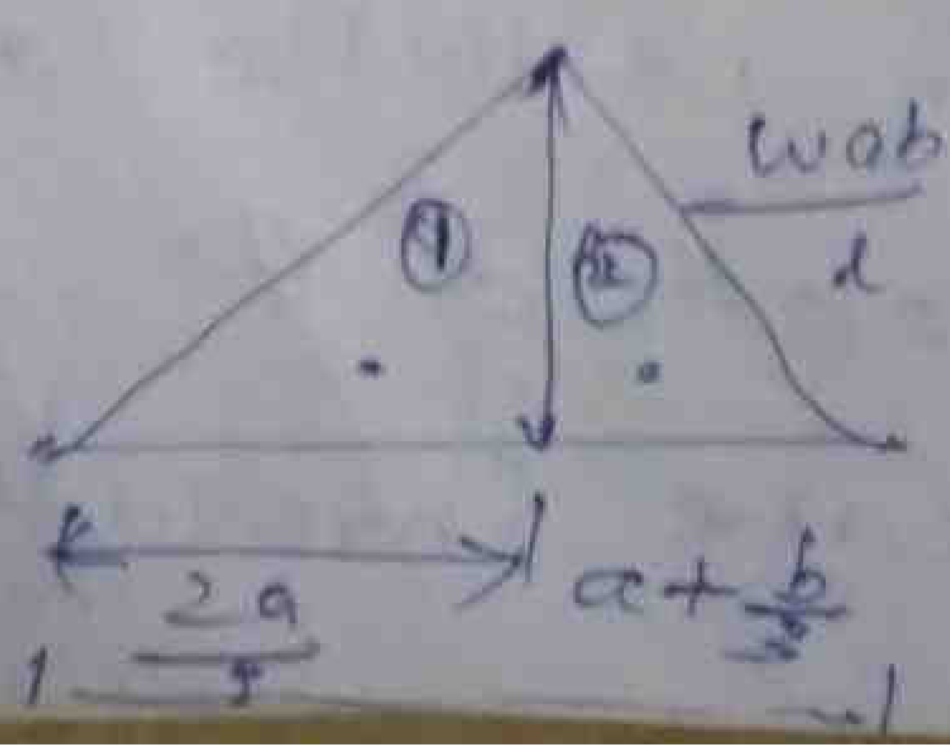


fixed beam.

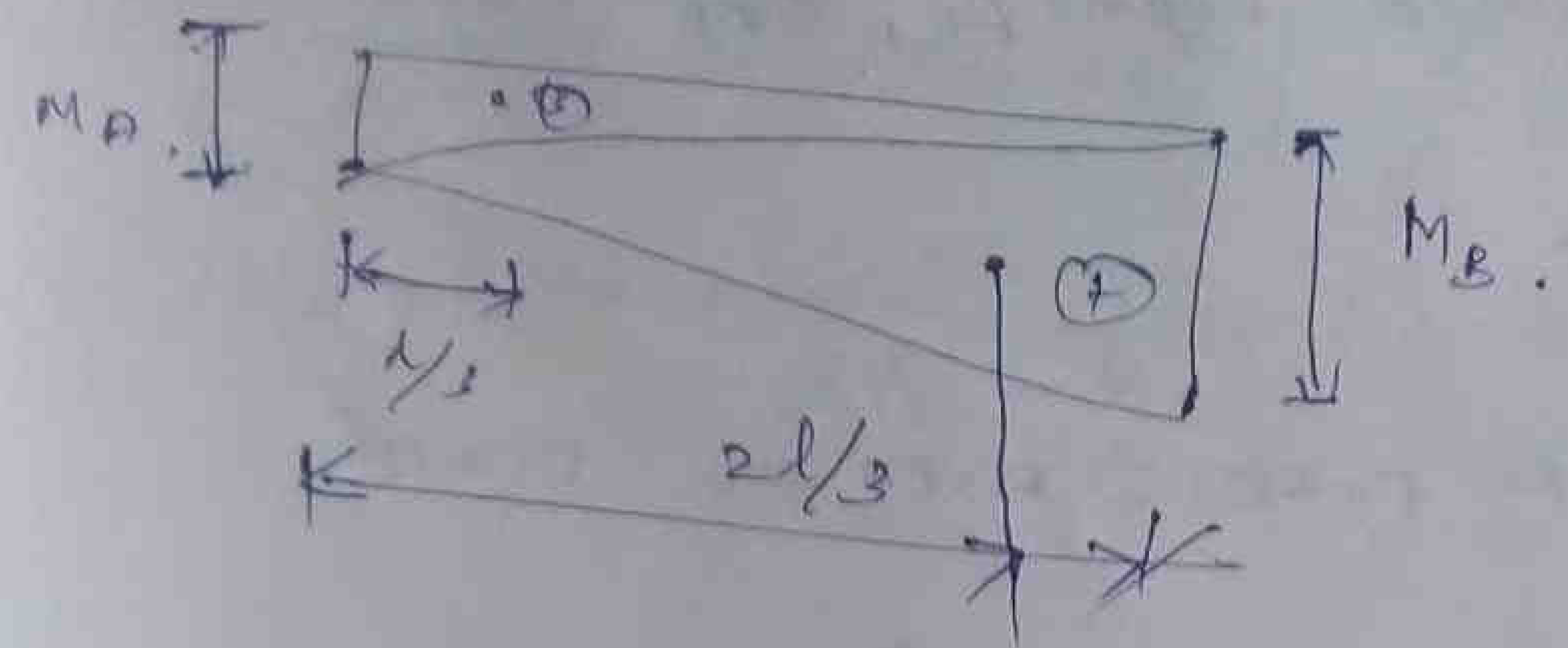




(b) Equivalent simply supported beam



Free BMD.



2) Free BMD

Area of Free BMD.

$$a_1 = \frac{1}{2} \times a \times \frac{w a^2 b}{2 l} = \frac{w a^2 b}{2 l}$$

$$a_2 = \frac{1}{2} \times b \times \frac{w a b}{l} = \frac{w a b^2}{2 l}$$

Centrical distance from A

$$x_1 A = \frac{2a}{3} \quad \& \quad x_2 A = a + \frac{b}{3} = \frac{3a+b}{3}$$

Fixed BMD

Apply redundent moments M_A & M_B to simply supported beam at

Area of fixed BMD A & B.

$$a_3 = \frac{1}{2} \times l \times M_A = \frac{M_A l}{2}$$

$$a_4 = \frac{1}{2} \times l \times M_B = \frac{M_B l}{2}$$

Centroidal distance from A

$$x_3 A = \frac{l}{3} \quad \& \quad x_4 A = \frac{2l}{3}$$

For Fixed beam AB, $\theta_{AB} = 0$.

$a_{AB} = 0$.

Area of Free = Area of Fixed

BMD = BMD.

$$a_1 + a_2 = a_3 + a_4.$$

$$\frac{wa^2 b}{l} + \frac{wab^2}{l} = \frac{M_A L}{2} + \frac{M_B l}{2}$$

$$\frac{wab(a+b)}{2l} = \frac{l}{2}(M_A + M_B).$$

$$M_A + M_B = \frac{wab}{l}.$$

$a_{AB}\ x_A = 0$ (or) $y_{AB} = 0$.

$$a_1 x_{1A} + a_2 x_{2A} - a_3 x_{3A} - a_4 x_{4A} = 0.$$

$$\left[\frac{wa^2 b}{2l} \times \frac{2a}{3}\right] + \left[\frac{wab^2}{2l} \times \left[\frac{3a+b}{3}\right]\right]$$

$$- \left[\frac{M_A l}{2} \times \frac{l}{3}\right] - \left[\frac{M_B l}{2} \times \frac{2l}{3}\right].$$

(or) $\frac{wab}{6l}\left[2a^2+3ab+b^2\right]-\frac{l^2}{6}\left[M_A+2M_B\right]=0.$

(or) $M_A+2M_B=\frac{wab}{l^3}\left[2a(a-b)+b(a+b+b(a+b))\right].$

$$=\frac{wab}{l^3}(a+b(2a+b)).$$

$$=\frac{wab}{l^3}\left[l(l+a)\right].$$

$$=\frac{wab}{l^2}(l+a).$$

4) Fixed end moments M_A and M_B (redundant reactions).

substract (1) from (2).

$$M_B=\frac{wab}{l^2}(l+a)-\left[\frac{wab}{l}\right]=\frac{wab}{l^2}$$

$$=\frac{wa^2b}{l^2}\Big].$$

substitute M_B in (1)

$$M_A+\frac{wa^2b}{l^2}=\frac{wab}{l}$$

(or)

$$M_A=\frac{wab}{l}-\frac{wa^2b}{l^2}=\frac{wab}{l^2}$$

$(l-a) = \frac{wab}{l^2} (b) = \frac{wab^2}{l^2}$

$MA = \frac{wab^2}{l^2}$ $MB = \frac{wa^2b}{l^2}$

Both MA and MB are hogging.

5) <u>vertical reactions V_A and V_B.</u>

Take moments about A.

$+(V_B \times l) - Wa - M_B + M_A = 0.$

(or) $V_B l - Wa - \frac{wa^2b}{l^2} + \frac{wab}{l^2} = 0.$

$\therefore V_B l = \frac{Wa}{l^2}\left[l^2 \; lab - b^2\right].$

$V_B = \frac{Wa}{l^3}\left[(l+b)(l-b) + ab)\right].$

$= \frac{wa}{l^2}\left[(l+b)\; a + 2ab\right] = \frac{wa^2}{l^3}(l+ab)\Big].$

From $\Sigma V = 0$.

$$V_A = W - V_B = W = \frac{Wa^2}{l^3}(l + 2b) = \frac{W}{l^3}$$

$$(l^3 - a^2 l - 2a^2 b).$$

$$= \frac{W}{l^3}\left[(a^3 + b^3 + 3a^2 b + 3ab^2) - (a^3 - a^2 b) - 2a^2 b)\right].$$

$$= \frac{W}{l^3}\left[a^3 + b^3 + 3a^2 b + 3ab^2 - a^3 - 3a^2 b\right] = 0.$$

$$= \frac{W}{l^3}(b^3 + 3ab^2) = \frac{Wb^2}{l^3}(b + 3a) \uparrow$$

$$= \frac{Wb^2}{l^3}(l + 2a) \uparrow$$

$$V_A = \frac{Wb^2}{l^3}(l + 2a) \uparrow$$

$$V_B = \frac{Wa^2}{l^3}(l + 2b) \uparrow$$

6) SF diagram SFD as shown in fig 2. a (b)

7) Final BMD

BM at B. $M_B = \frac{wa^2b}{l^2}$

BM at C.

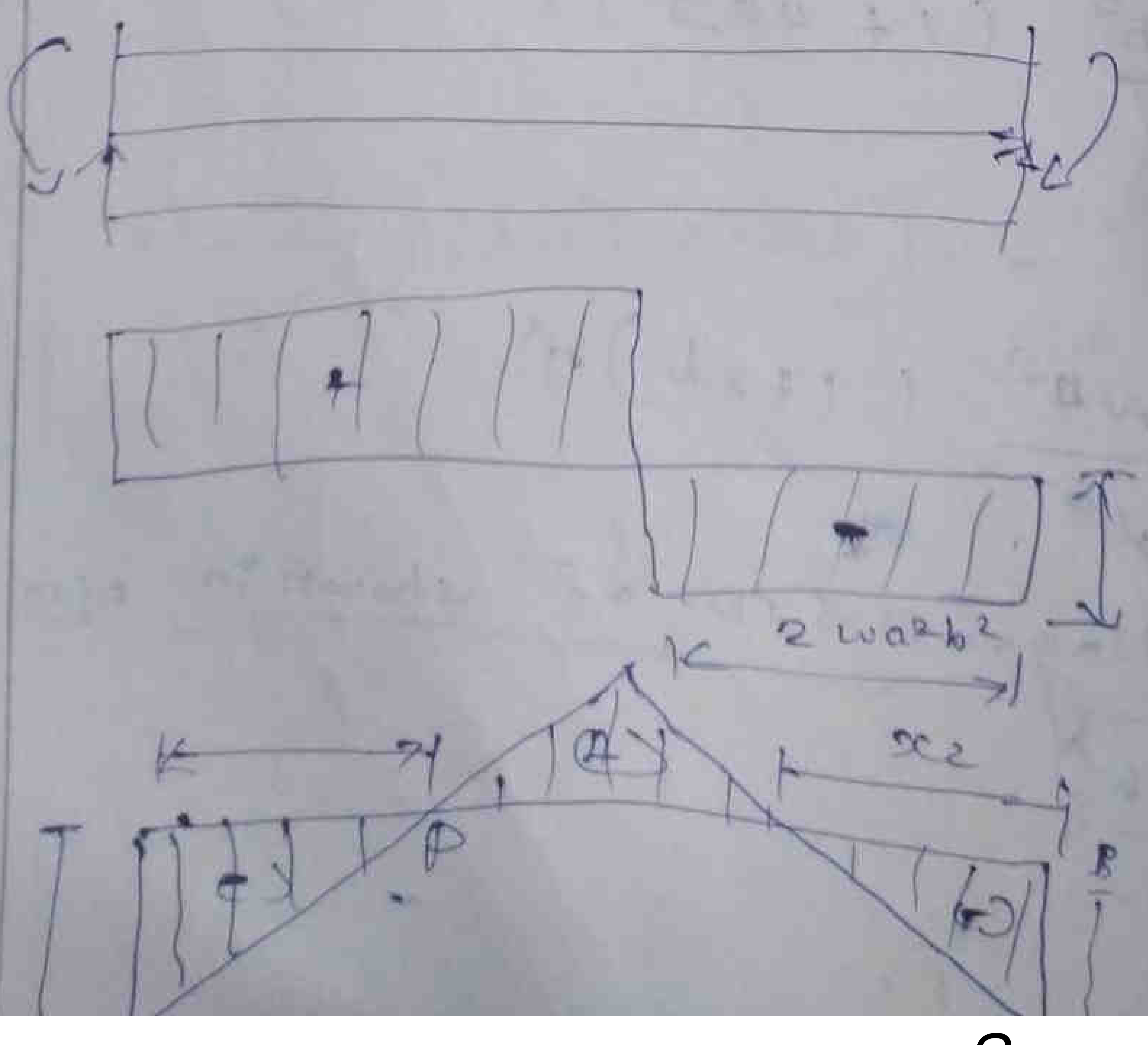
$$M_C = \frac{wa^2b}{l^2} + V_B \times b.$$

$$= \frac{-wa^2b}{l^2} + \frac{wa^2}{l^3}(l+2b)b$$

$$= \frac{+2wa^2b^2}{l^3}$$

$$M_A = \frac{-wab^2}{l^2}$$

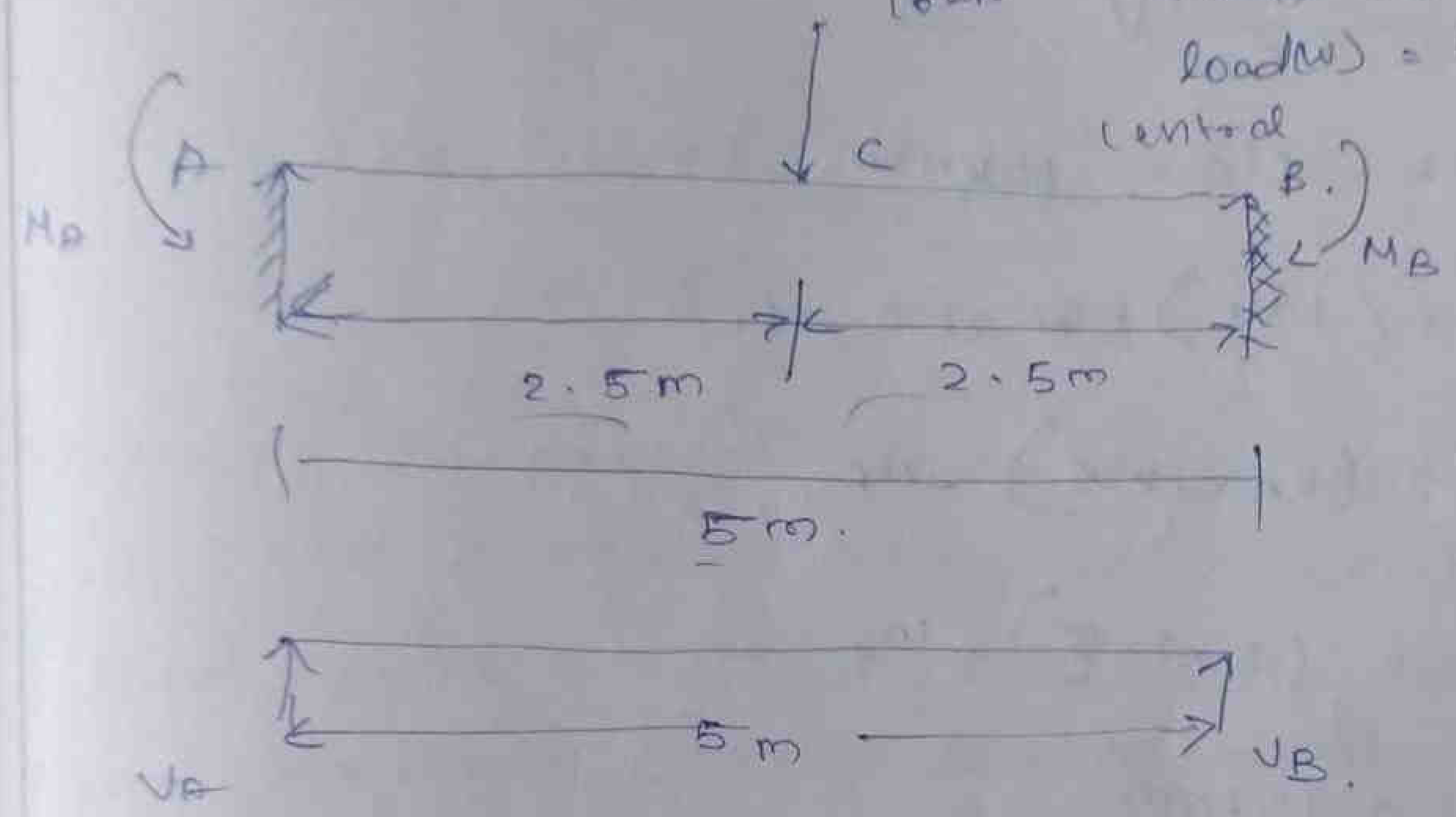
Find BMD is shown Fig



$\frac{wa^2b}{}$

A Fixed beam of 5m carries Central beam point load of 16 kN. Determine the Fixed moment maximum moment, Sketch the SFD, Sketch the BMD and mark the point of contraflexure structure. Find the maximum central deflection.

Given l = 5m
load(W) = 16kN
central



Sol

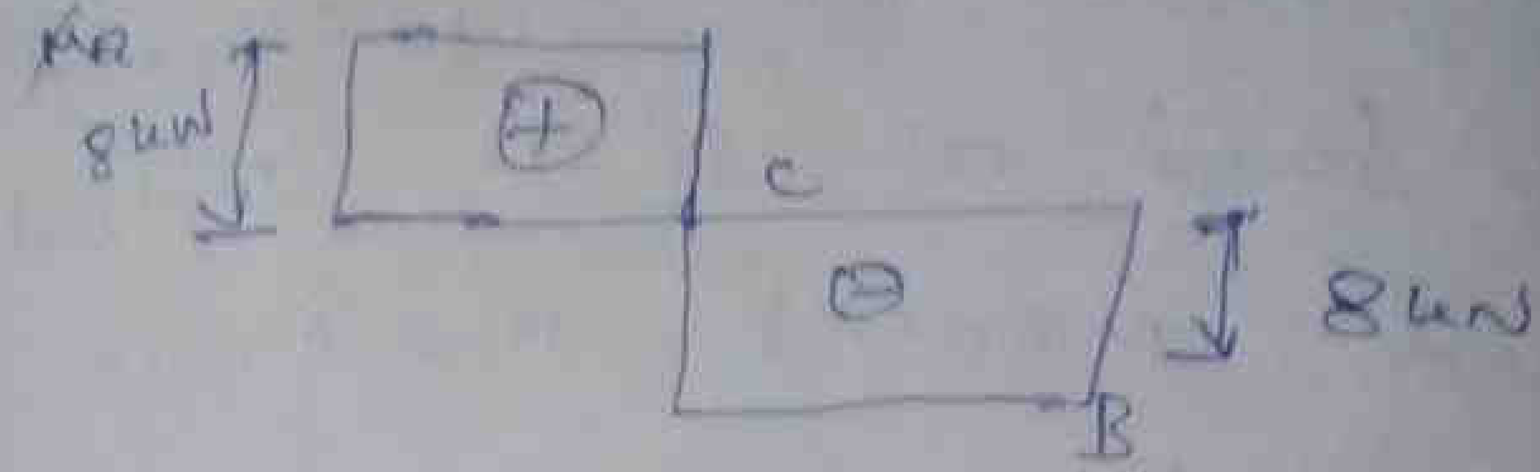
① Fixing moment $= M_A = M_B = \dfrac{Wl}{8}$

$$= \frac{16 \times 5}{8} = 10\ kNm$$

② vertical support reactions

$$V_A = V_B = \frac{W}{2}$$

$$= \frac{16}{2} = 8\ kNm$$



③ Shear force diagram

④ Final Bending moment

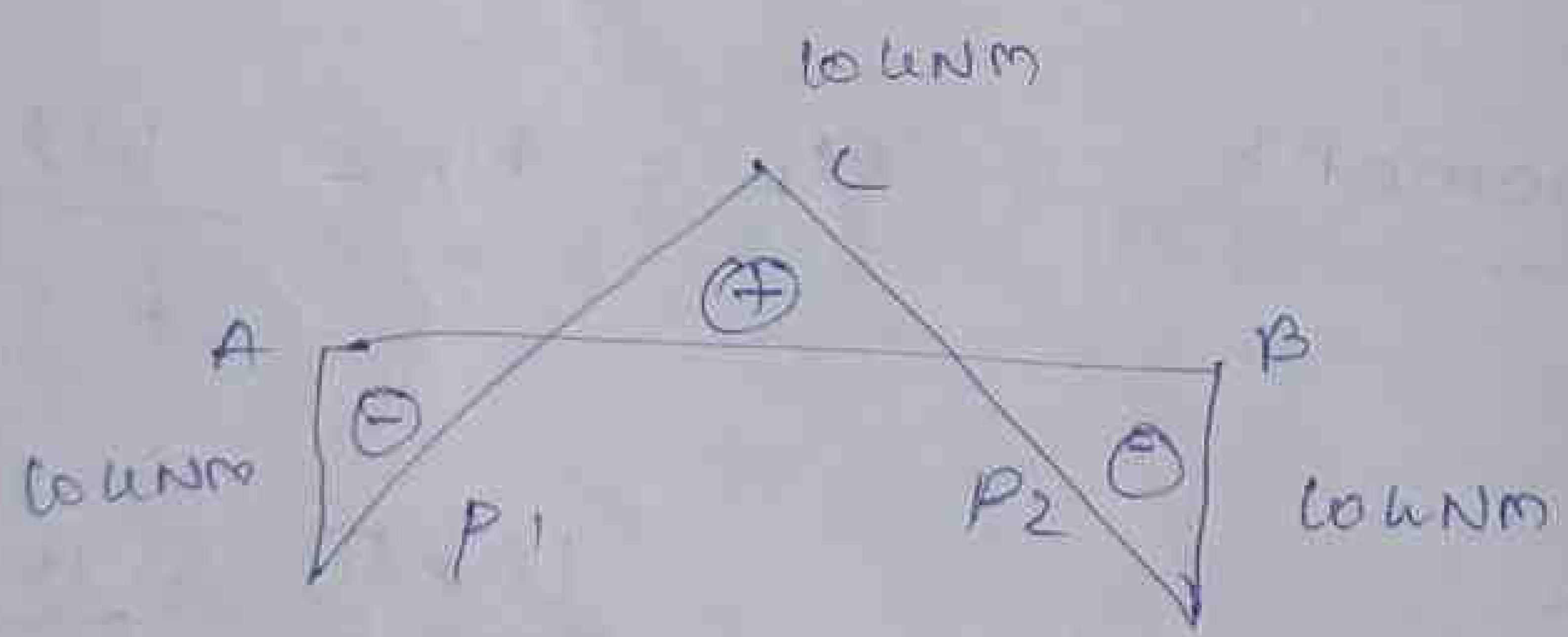
$M_A = M_B = 10\text{kNm}.$

Max (+ve) BM at C, M_C

$M_C = (V_A \times 2.5) - M_A.$

$= (8 \times 2.5) - 10$

$= \underline{10\text{kNm}}$



④ Final BMD

⑤ point of contraflexure (P_1 & P_2)

BM at $P_1 = V_A \times X - M_A$

$= \cancel{2.5}.$

$= 8 \times x - 10$

$= 10/8$

$= \underline{1.25\text{m}}$

Due to symmetry point of contraflexures P_1 & P_2 are at 1.25m from the supports.

(b) maximum central deflection y_c

$$y_c = \frac{wl^3}{192 EI}$$

$$= \frac{16 \times 5^3}{192 EI} \quad y_c = \frac{10.417}{192 EI}.$$

A simply supported beam of a uniform c/s and span 6m carry is a central load point w. i) calculate value of w. and maximum sagging bending moment if the ii) central deflection in respected 1/300 of span. iii) If the simply supporte ends are replaceby fixed supports, whats is a value of central load when a deflection in both cases? b) maximum positive BMD is same in both cases

Assume $EI = 1.8 \times 10^3 \, kNm^2$.

Given data:

simply supported beam

span $(l) = 6m$

case-1 deflection $y = 1/300$ span.

$EI = 1.8 \times 10^3 \, kNm^2$.

Find:

(i) Calculate the value of w.

(ii) simply supported beam.

(2) simply supports supported replaced by fixed support.

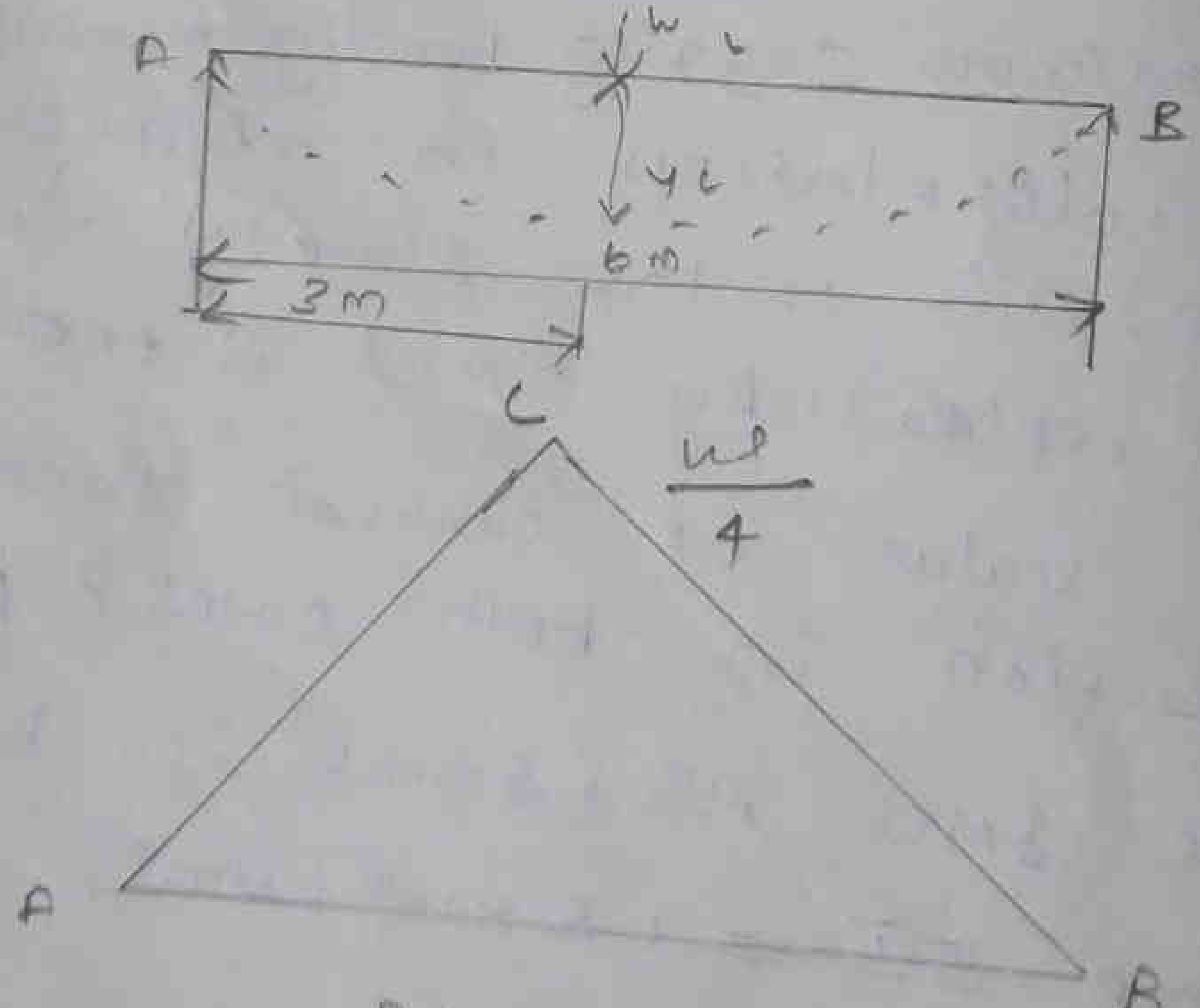
(a) defflection

(b) maximum positive BM at both cases.

Solution:

(1) Deflection $y = \frac{1}{300} \times 6$.

$= \underline{0.02m}$.



BMD.

max central deflection

$$\frac{Wl^3}{48EI} = \frac{W \times (6)^3}{48 \times 1.8 \times 10^3}$$

$$= \frac{8.100}{48 \times 1.8 \times 10^8} \quad \underline{W = 2.5 \times 10^{-3}}.$$

Equate max central deflection $\Big\}$ = allowable (yc). deflection.

$$= \frac{0.02}{}$$

$$w \times 2.5 \times 10^{-3} = 0.02$$

$$w = \frac{0.02}{5 \times 10^{-3}}$$

$$\boxed{W = 8 kN}$$

$$BMD = \frac{wl}{4}$$

$$= \frac{8 \times 6}{4}$$

$$= \frac{48}{4} = 12 \text{ kN}$$

$$\boxed{= 12 \text{ kN}}$$

2) Beam with fixed ends.

allowable deflection $\Big\}$ = 0.02 m.

Span $l = 6$ m.

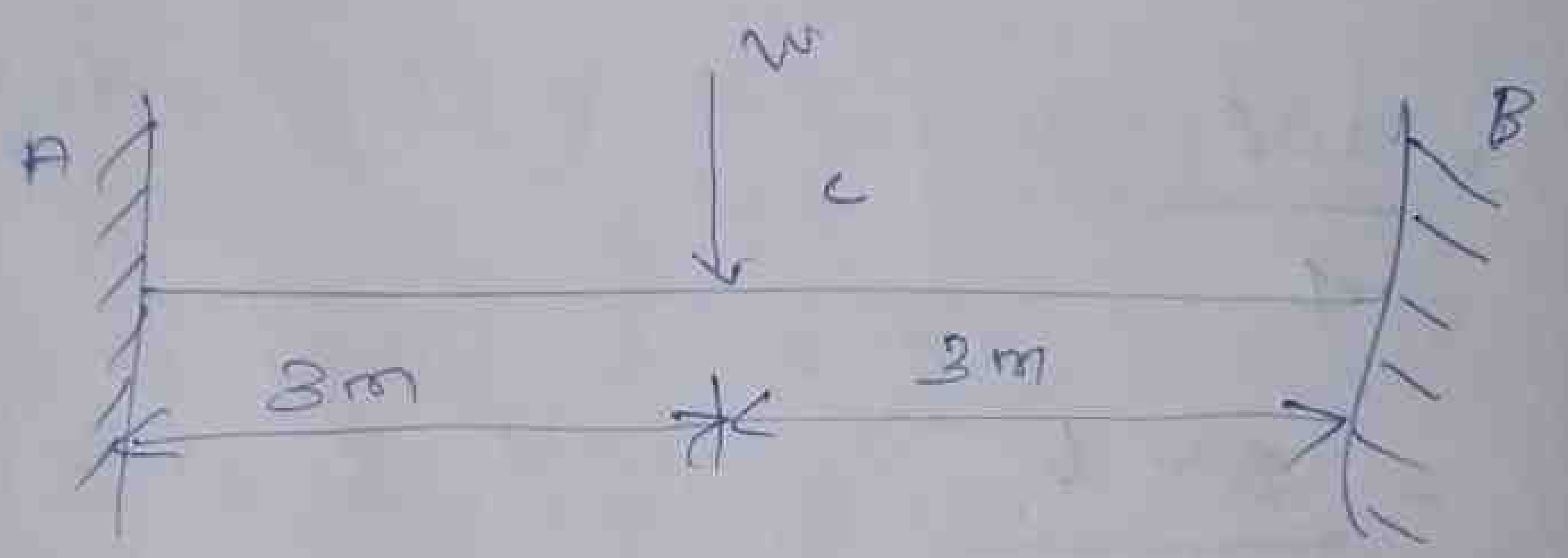
max. central deflection of a fixed beam $\Big\}$ $= \frac{wl^3}{192EI}$

$$= \frac{w \times 6^3}{192 \times 1.8 \times 10^3} = \frac{0.02 \times (6)^3}{192 \times 1.8 \times 10^3}$$

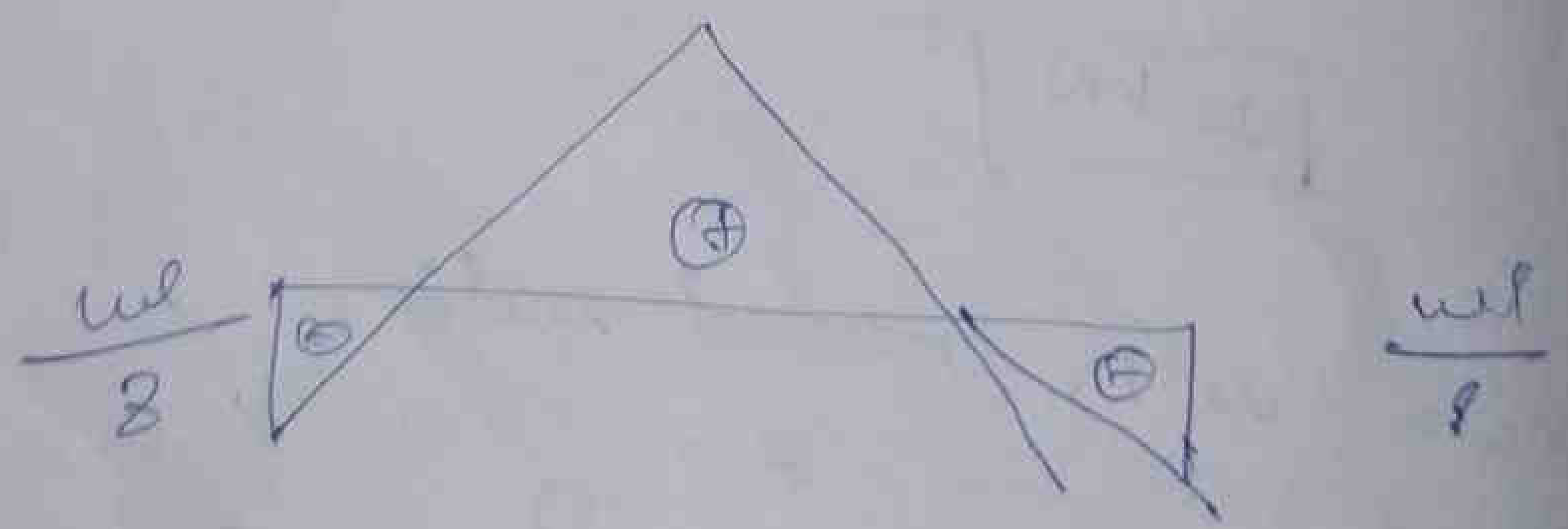
Equate allowable & max:

$w = 32\,kN$

(b) when maximum (+ve) B.M is same for both cases.



Fixed Beam.



Fixed beam

$V_A = V_B = \frac{W}{2}$

$M_A = M_B = \frac{-Wl}{8}$

max $= (+ve)$ BM

$= -M_A \left(\frac{W}{2} \times \frac{l}{2} \right)$

$= \frac{W \times 6}{8} \times \frac{W \times 6}{4}.$

$= 0.75W \times 1.5W$

$= \underline{1.125W}$

$= \cancel{36kN}$

$y = W \times 1.125$

$0.02 = W \times 1.125$

$W = \underline{0.02/1.125}$

$W = \underline{0.17 kN}$

Allowable $(+ve)$ BM in fixed beam

$=$ max $(+ve)$ from SS beam

$= 12 kNm$

$W = \underline{0.17 kN}$

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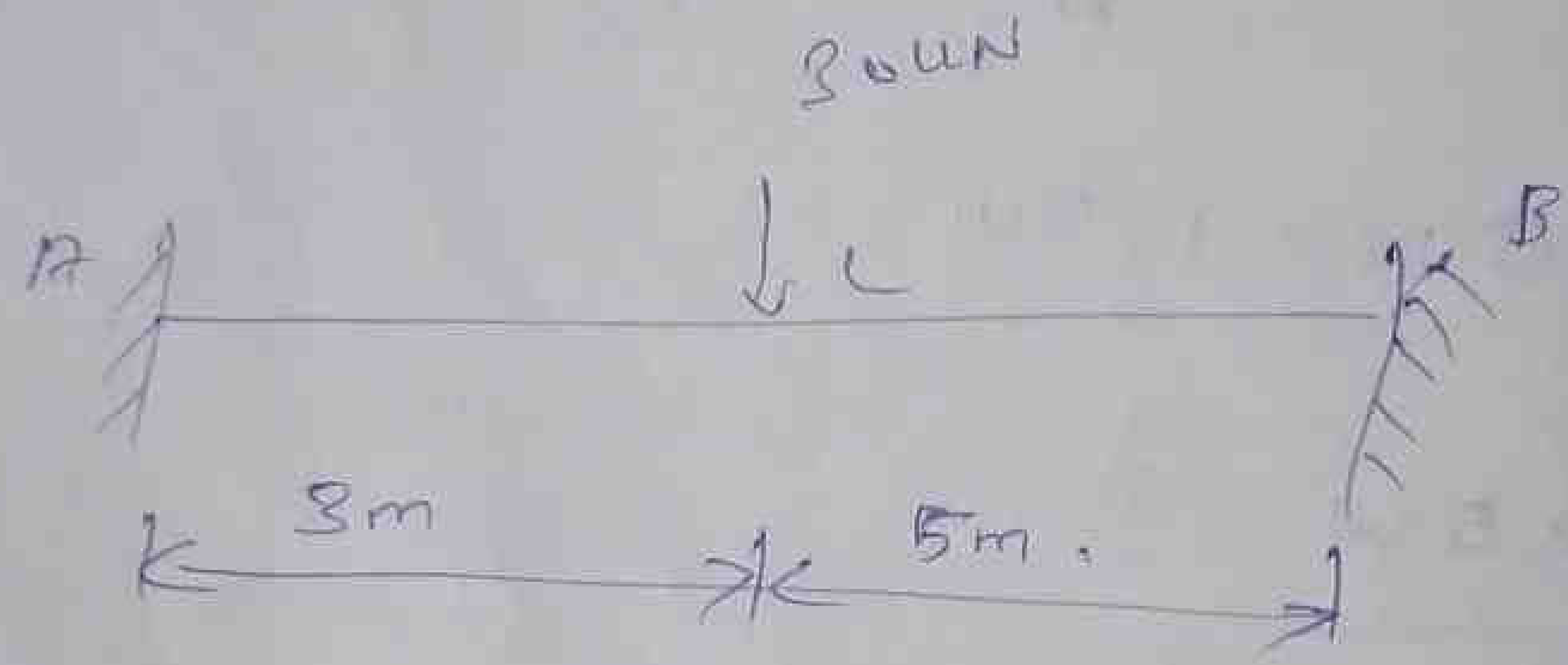
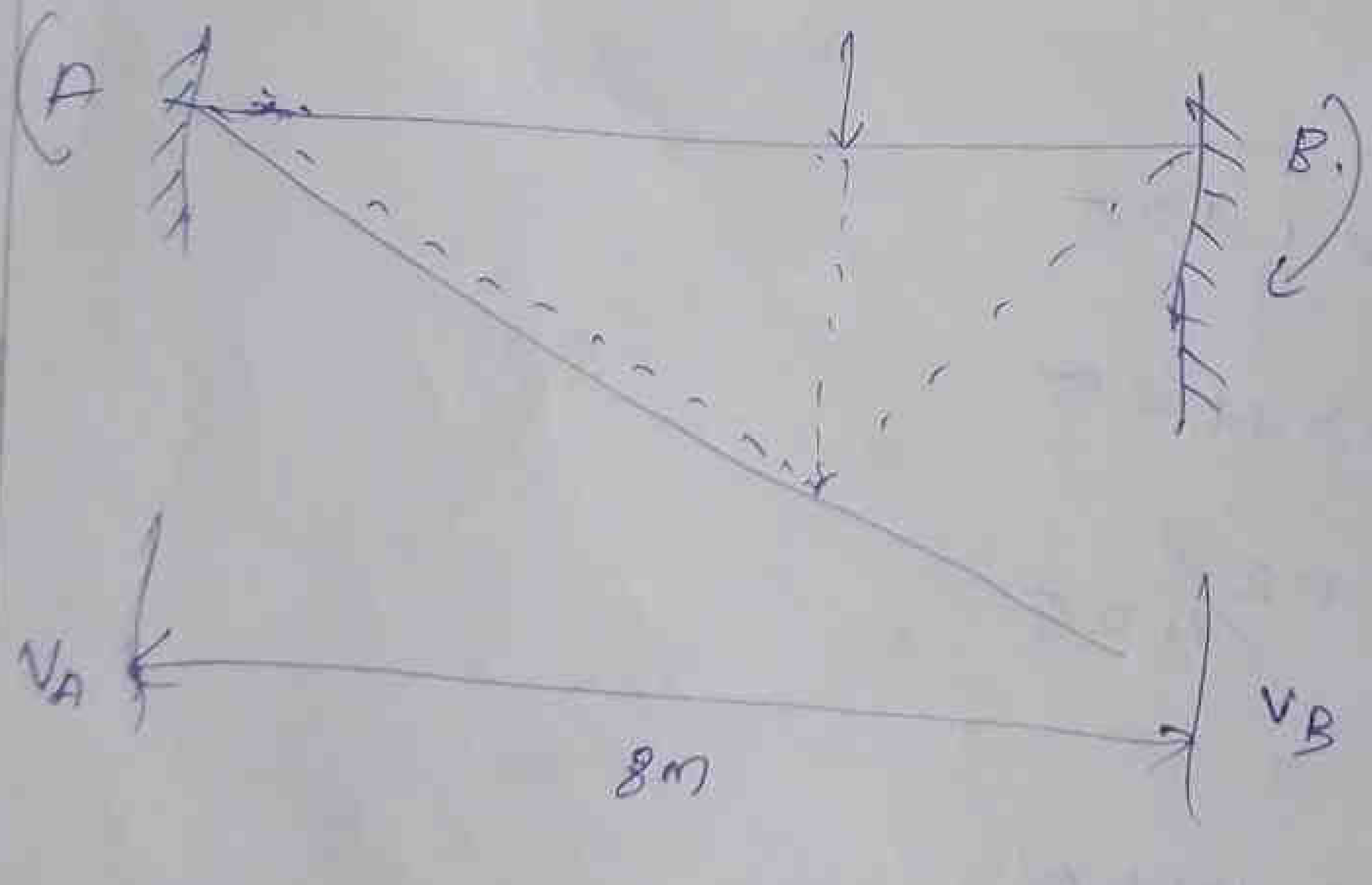
A Fixed beam of 8m span is subjected to a concentrated load of 30kN acting at 3m from left support as shown in Figure (a) determine i) Support moment and reaction ii) Change of slope of a and c use mohr's area theorem to find the slope $EI = 1 \times 10^5\ kNm^2$

Given data:





Given data.

a Fixed beam of = 8m

load of (l) = 30kN

acting at = 3m

$EI = 1 \times 10^5\ kN-m^2$

Data:

$a = 3m$, $b = 5m$, $l = 8m$.

$EI = 1\times10^5 \ kNm^2$

Solution:

Support moments

$$M_A = \frac{wab^2}{l^2}$$

$$= \frac{30\times3\times5^2}{8^2}$$

$$= 35.156 \ kN.m.$$

$$M_B = \frac{wa^2b}{l^2}$$

$$= \frac{30\times3^2\times5}{8^2}$$

$$= 21.094 \ kN.M.$$

M_A and M_B are hogging.

moments.

2) Support reactions V_A V_B

Take moments about A

$$+(V_B\times8) - (30\times3) - M_B + M_A = 0 \ (or)$$

$$8V_B = 90 - 21.094 + 35.156 = 0 \ (or)$$

$$8V_B = 75.938$$

$$V_B = 9.492 \ kN \uparrow$$

3) Change of slope b/w A & B.

beam AB as an equivalent cantilever beam. Draw the elastic curve to the equivalent cantilever beam.

$$\theta_{AC} = \frac{a_{AC}}{EI}$$

Consider BMD in b/w A & C.

4) BMD by parts

(i) Due to $V_A = 20.578$ kN

BM at $A = 0$.

BM at $C = + V_A \cdot AB$.

$= + 20.508 \times 3 = \underline{61.524}$ kN.m

area $a_1 = \frac{1}{2} \times 3 \times 61.524$

$= 92.286$ kN-m.

(ii) due to M_A: 35.156 kN-m

BM at A = 35.156 kNm.

BM at C = 35.156 kN-m.

Area $a_2 = 3 \times 35.156$

$= 105.468$ kN-m^2

$a_{AC} = a_1 - a_2$

$= 92.286 - 105.468$

$= 13.182 \text{ kN-m}^2$,

(iii) change of slope b/w A & C.

$$\theta_{AC} = \frac{a_{AC}}{EI} = \frac{13.182}{1 \times 10^5}$$

$$= 1.32 \times 10^{-4} \text{ rad.}$$

result!

1) support moment M_A

(i) At A = 35.156 kN-m.

(ii) At B = 21.094 kN-m.

2) support reactions.

(i) At A V_A = 30.508 kN ↑

(ii) At B V_B = 9.492 kN ↑

3) change of slope b/w A and C

$\theta_{AC} = 1.32 \times 10^{-4}$ rad

Calculate the support ... moment of a fixed beam of span 10m Fixed at A and B carrying a udl of 8kN/m over the entire length and a point load of 50kN at 4m. For the supports B.

sol:

let V_B and M_B be reactions at B.

Equivalent Cantilever.

Remove the fixity at B.

Consider the Equivalent Cantilever subjected to V_B M_B W and udl as shown in Draw the BMD by parts.

② BMD areas and centroidal distance from A

$$a_1 = \frac{1}{2} \times 10 \times 10\ V_B.$$

$$= \underline{50 V_B}$$

$$x_1 = \frac{1}{3} \times 10 = \frac{10}{3}\ m.$$

$$a_2 = 10 \times M_B = 10 M_B.$$

$$x_2 = \frac{10}{3} = 5$$

$$a_3 = \frac{1}{3} \times 10 \times 400$$

$$= \frac{4000}{3}\ kN\text{-}m.$$

Calculate the support reactions and moment of a fixed beam of span 10m fixed at A and B carrying a udl of 8kN/m over the entire length and a point load of 50kN at 4m. For the support B

sol:

let V_B and M_B be reactions at B.

Equivalent cantilever.

Remove the fixity at B.

Consider the equivalent cantilever subjected to V_B M_B W and udl as shown in Draw the BMD by parts.

③ BMD areas and centroidal distance from A

$a_1 = \frac{1}{2} \times 10 \times 10\, V_B$.

$= \underline{50 V_B}$

$x_1 = \frac{1}{3} \times 10 = \frac{10}{3}$ m.

$a_2 = 10 \times M_B = 10 M_B$.

$x_2 = \frac{10}{3} = 5$

$a_3 = \frac{1}{3} \times 10 \times 400$

$= \frac{4000}{3}$ kN-M.

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$x_3 = \frac{1}{4} \times 10 = 2.5m$

$a_4 = \frac{1}{2} \times 6 \times 800 = 900 \, kN\text{-}m^2$

$x_4 = \frac{1}{3} \times 6 = \underline{2m}$

(3) Redundant reactions V_B and M_B.

since tangents at A and B coincide.

$\theta_{AB} = 0$.

$a_{AB} = a_1 + a_2 - a_3 - a_4 = 0$.

(or) $50 V_B - 10 M_B \frac{-4000}{3} = 900 = 0$ (or)

$d_{AB} = 0$ since tangent at A and B $= a_1 x_1 - a_2 x_2$

$a_3 x_3 - a_4 a_4 = 0$ (or).

$$\left[50 V_B \times \frac{10}{3}\right] - \left(10 M_B \times 5\right) - \frac{4000}{3} \times 2 = 0.$$

(or) $\frac{500}{3} V_B - 50 M_B - \frac{1000}{3} - 1800 = 0$.

(or) $500 V_B - 150 M_B - 1000 - 5400 = 0$.

$5 V_B - 5 M_B = 154 \quad \ldots \quad (6)$

$(1) - (2) \Rightarrow 0.5 M_B = 69.33$

(or) $M_B = 138.66$ kN (Hogging).

$5V_B = 138.66 = 223.33$

(or) $V_B = 72.4$ kN↑

From $\Sigma V = 0$. $V_A = 50 + (8 \times 10) - V_B$

$= 50 + 80 - 72.4 = 57.6$ kN↑

(4) Final moment:

$M_B = -138.66$ kN-M.

$M_C = -M_B + (V_B \times 4) - \frac{8 \times 4^2}{2}$

$= -128.76 + (72.4 \times 4)$

$= -8 \times \frac{4^2}{2} = 86.94$ kNM.

$M_A = 138.66 + (72.4 \times 10) \frac{8 \times 10^2}{2} - (50 \times 6)$

$= -114.66$ kN-M²

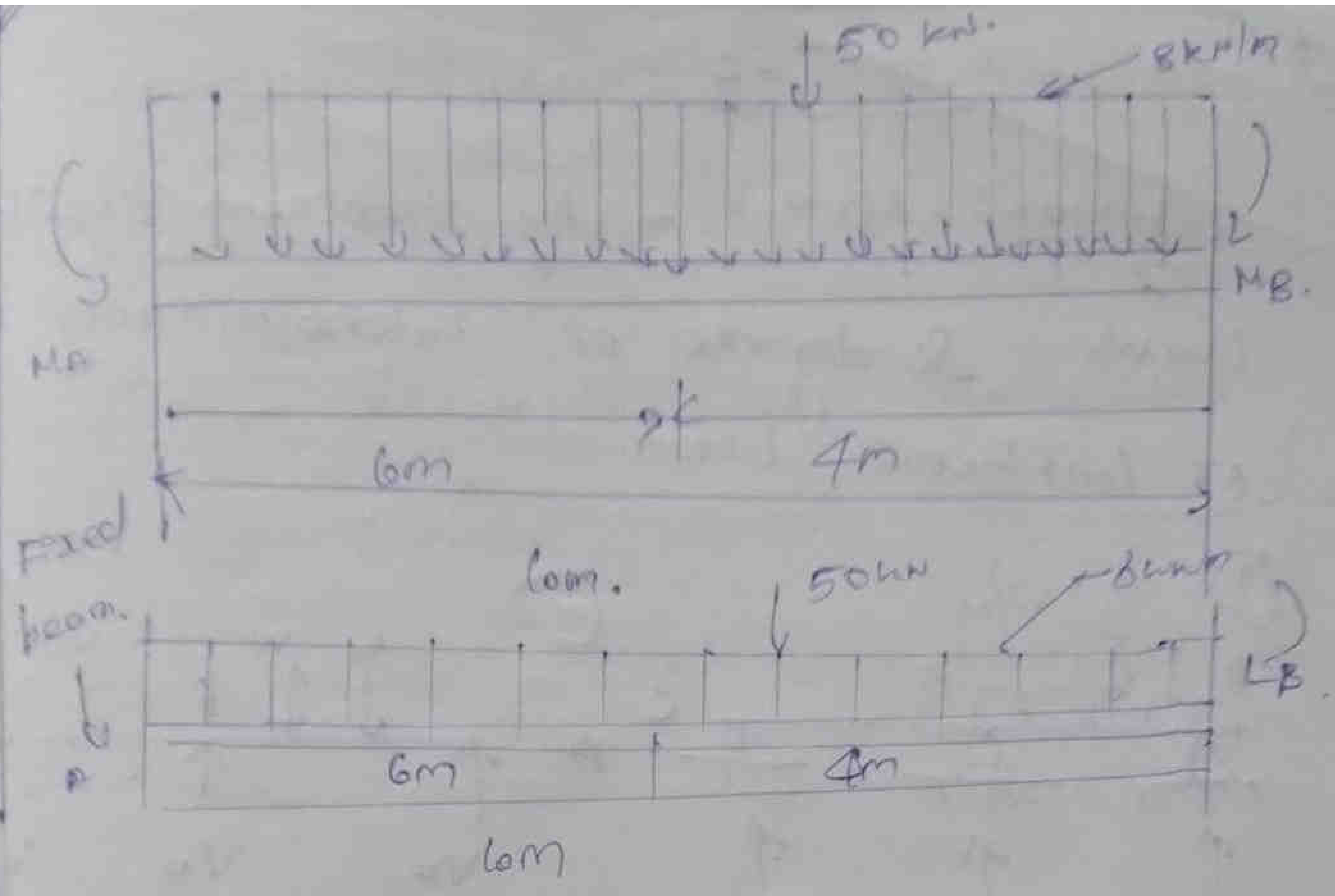
Result:

(1) Support reactions

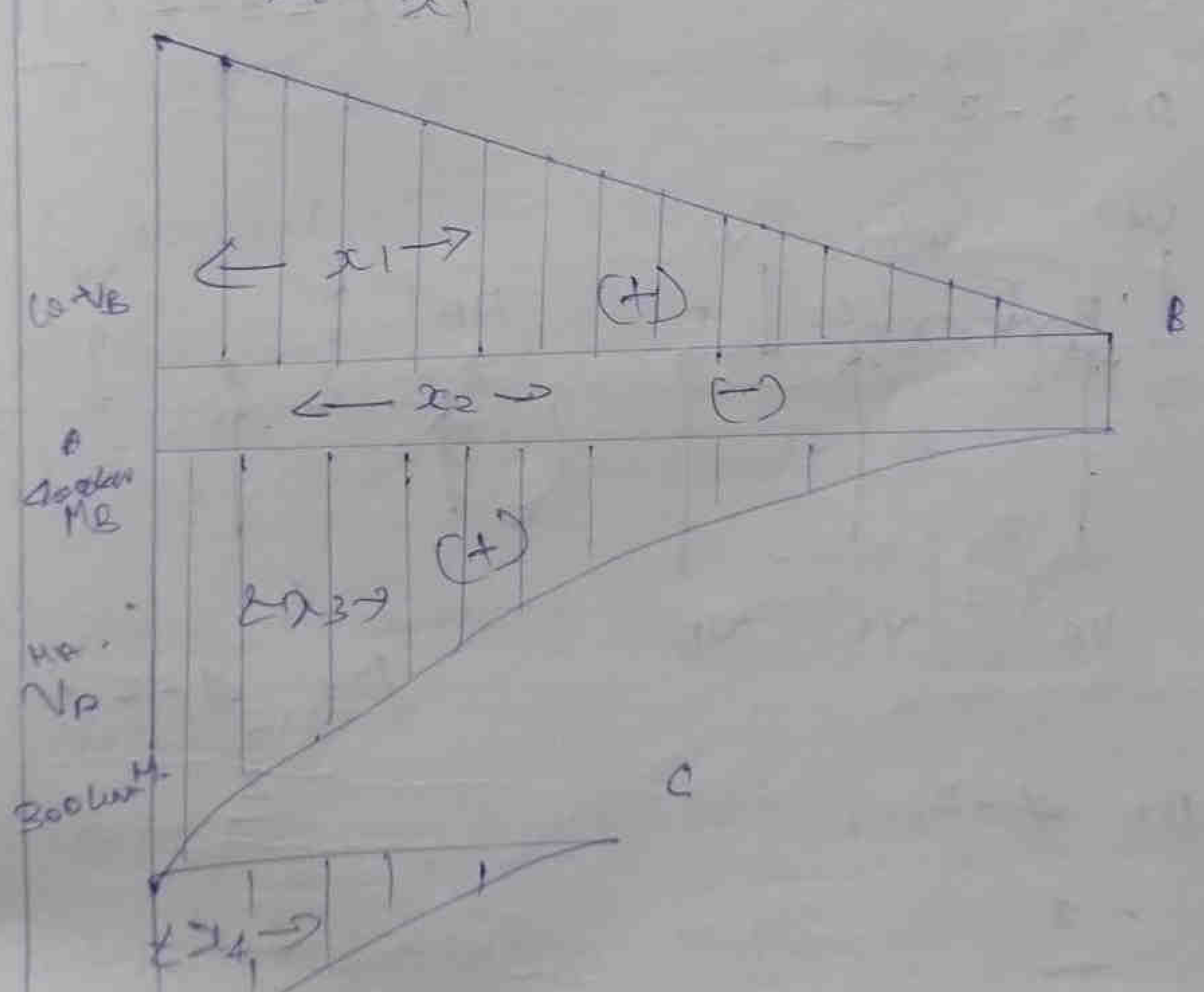
$V_A = 57.6$ kN↑ $V_B = 72.4$ kN↑

(2) Support moment.

$M_A = -114.66$ kN-M $= -138.66$ kNm.



Equivalent Cantilever.

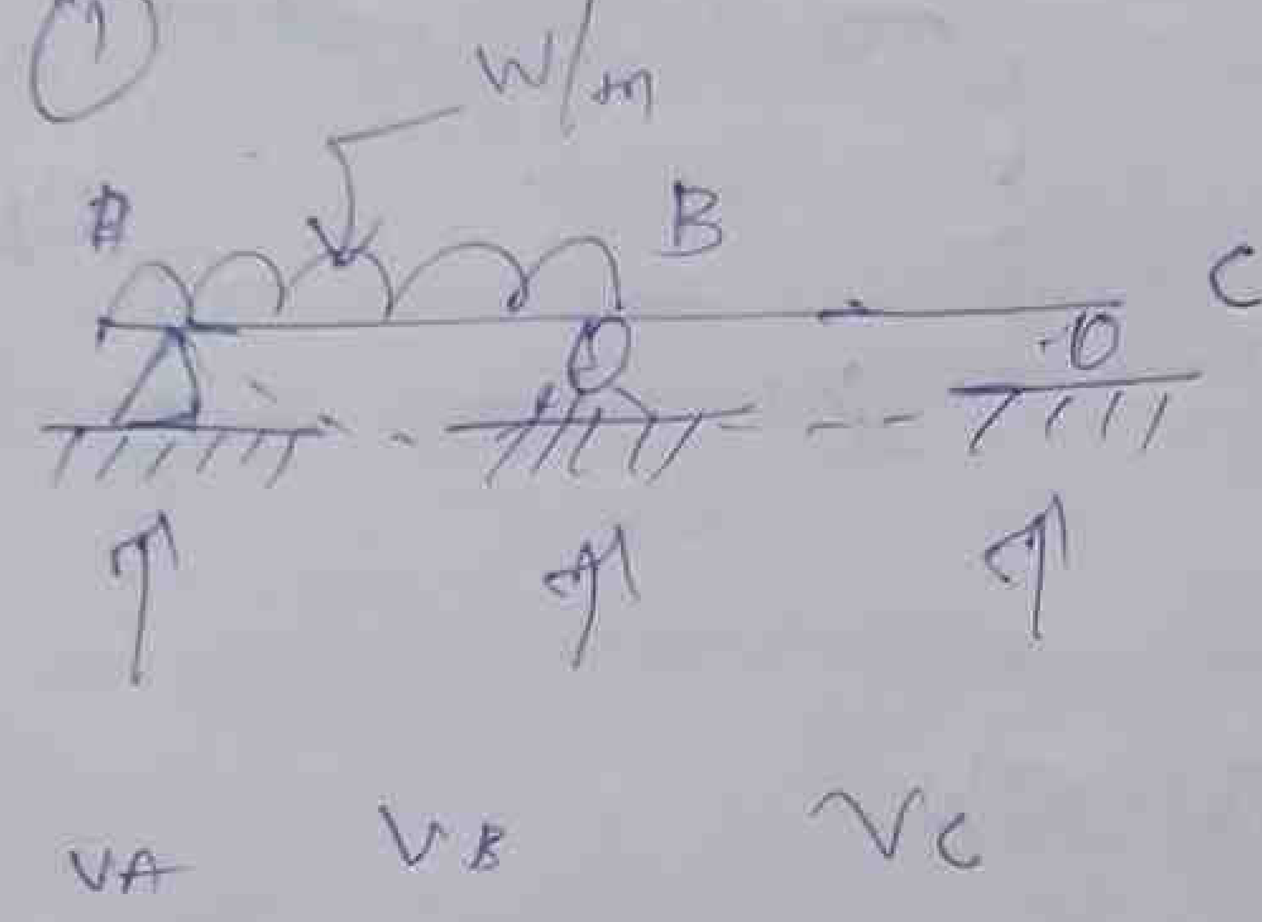


21.7.22.

UNIT - 2

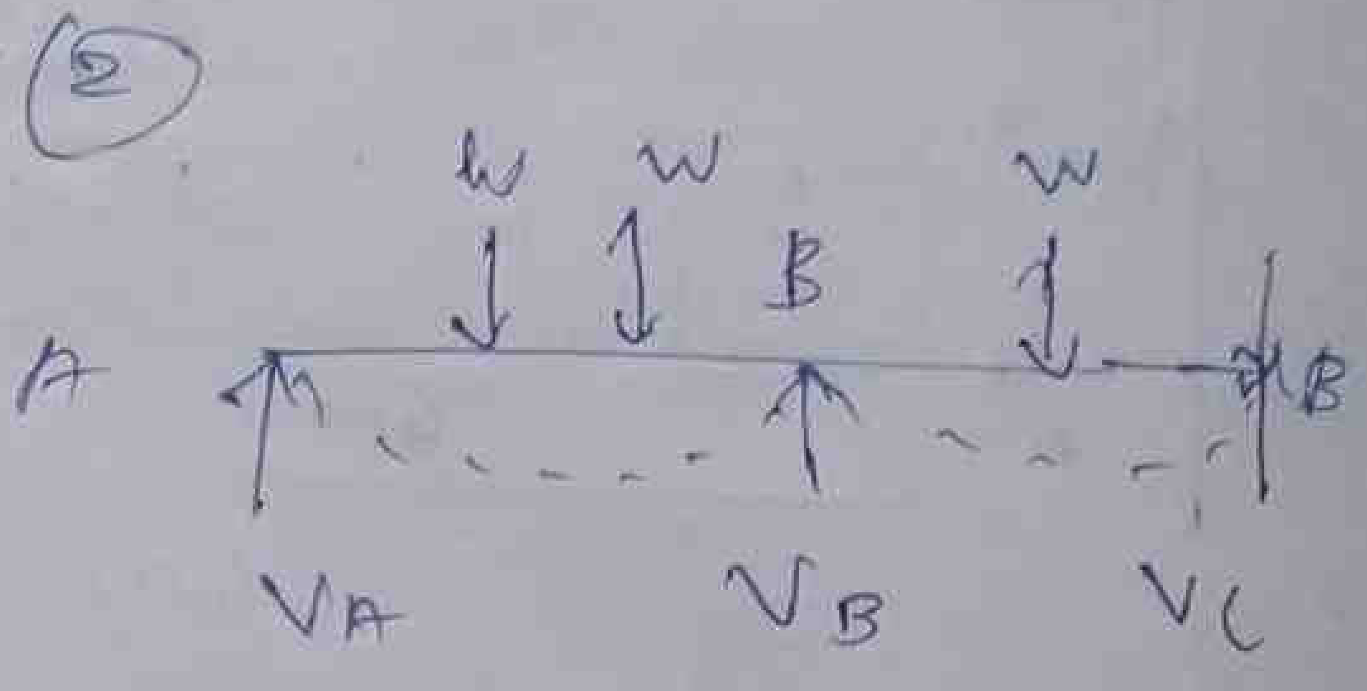
Continuous Beams - by theorem Elastic curve & degree of indeterminancy of continuous beam.

(1)



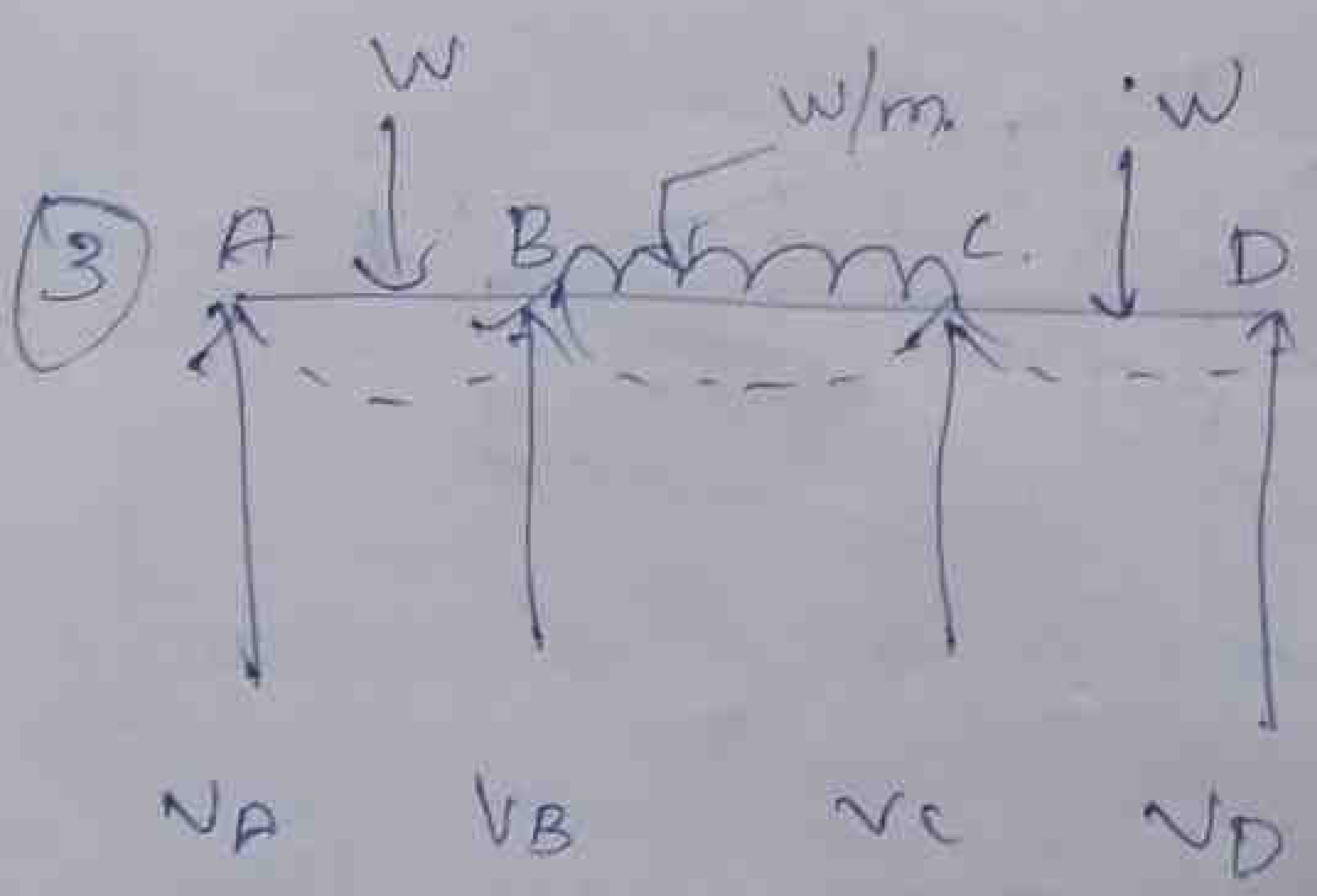
$D = 3 - 2 = 1$

(2)



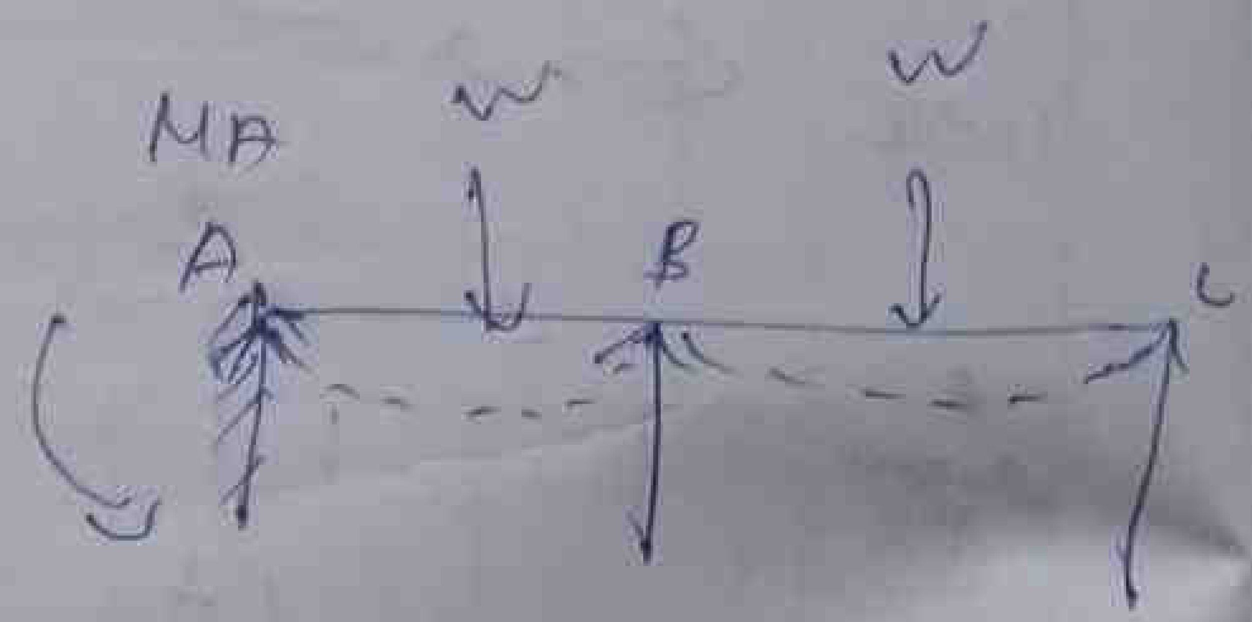
$D = 3 - 2 = 1$

(3)



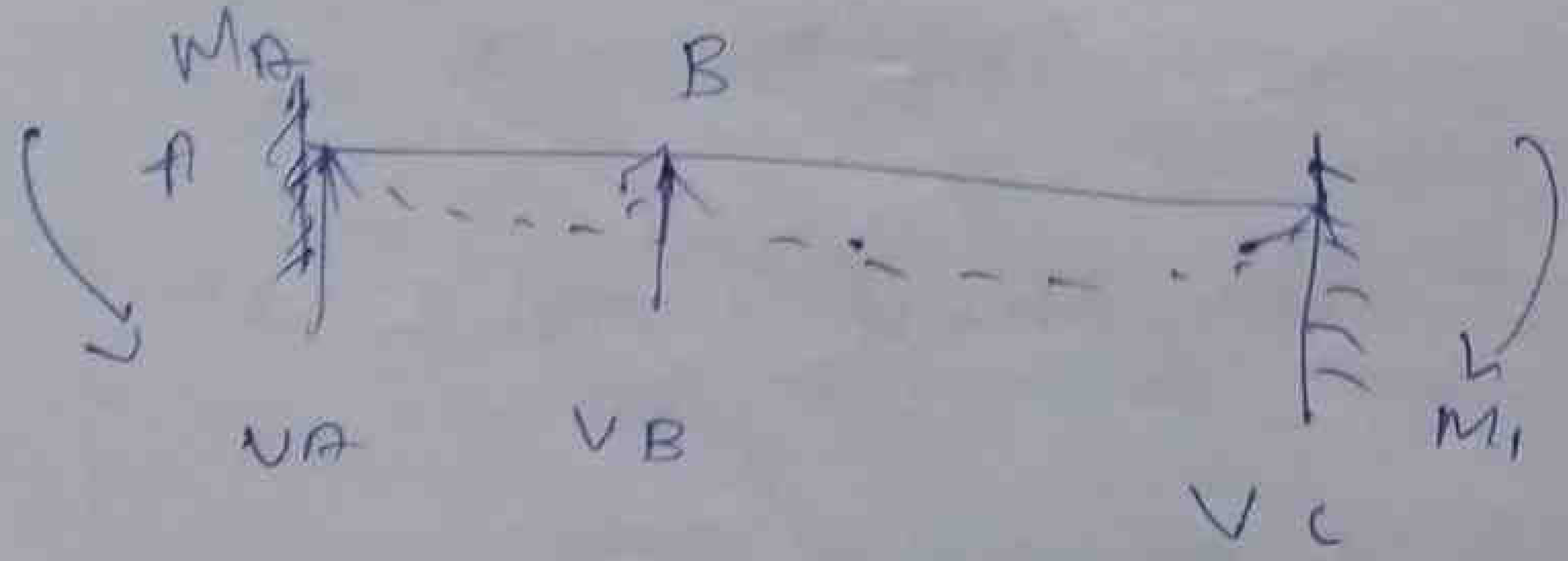
$D = 4 - 2$

$= 2$



$D = 4 - 2$

$= 2$



$D = 5 - 2 = 3$

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known equation Static Equilibrium equation ($\Sigma V=0$ & $\Sigma M=0$) are $\underline{2}$

unknown reaction (V_A, V_B & V_C) are $\underline{3}$ Hence Statically indeterminacy

D = unknown reaction - known reaction

= 3 - 2

= $\underline{1}$

Analysis of continuous beams.

1) Compatibility Methods.

2) equilibrium methods.

1) Compatibility Methods.

i) General method of consistent deformations.

ii) Clapeyron's three moment theorem.

iii) Column analogy Method.

iv) Elastic Centre method.

v) Maxwell - Mohr's method.

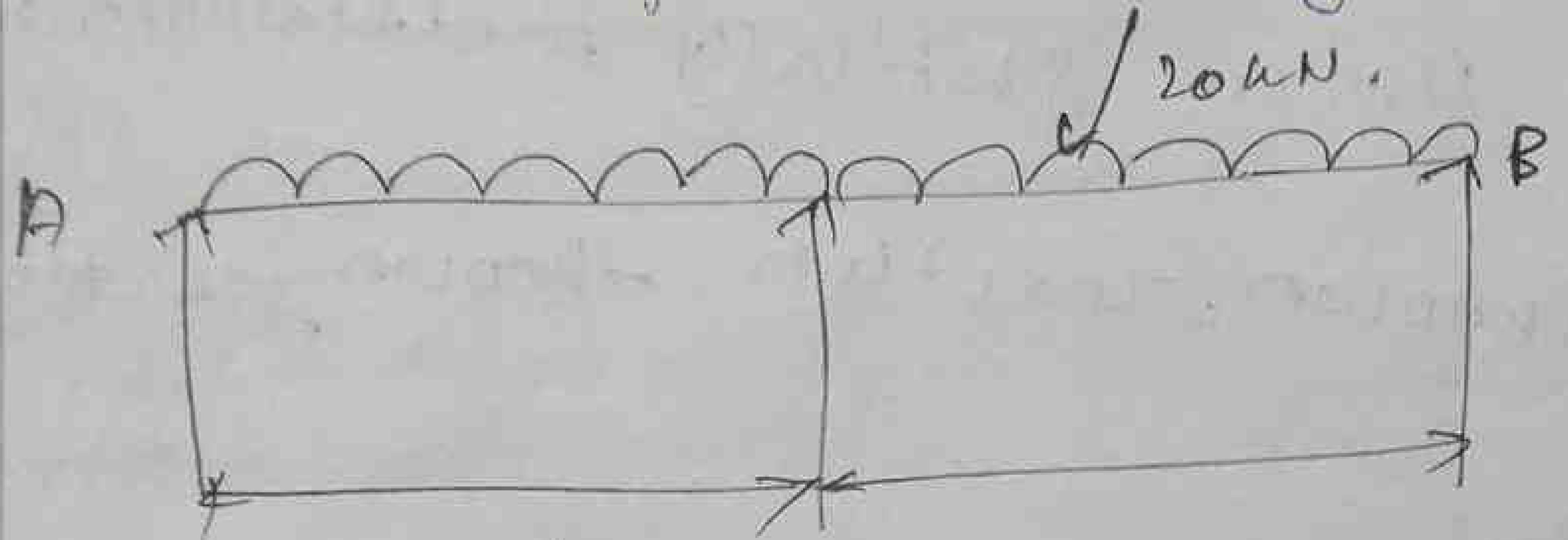
vi) Castigliano's strain energy method.

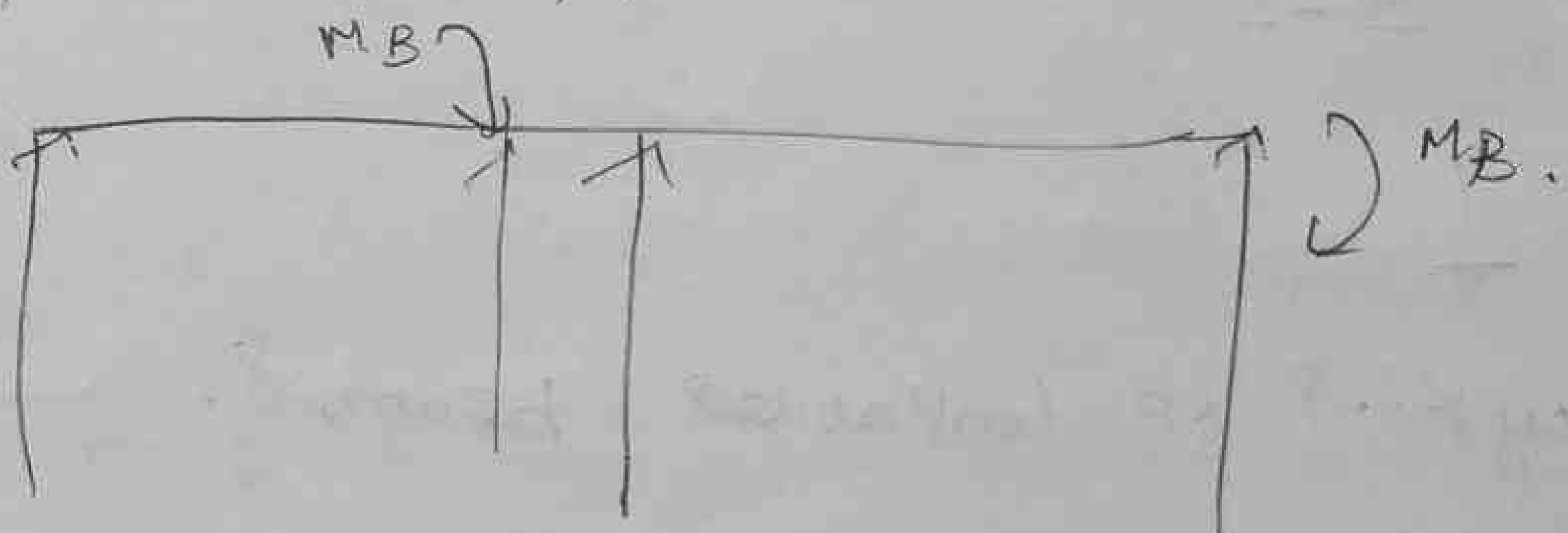
2) equilibrium methods.

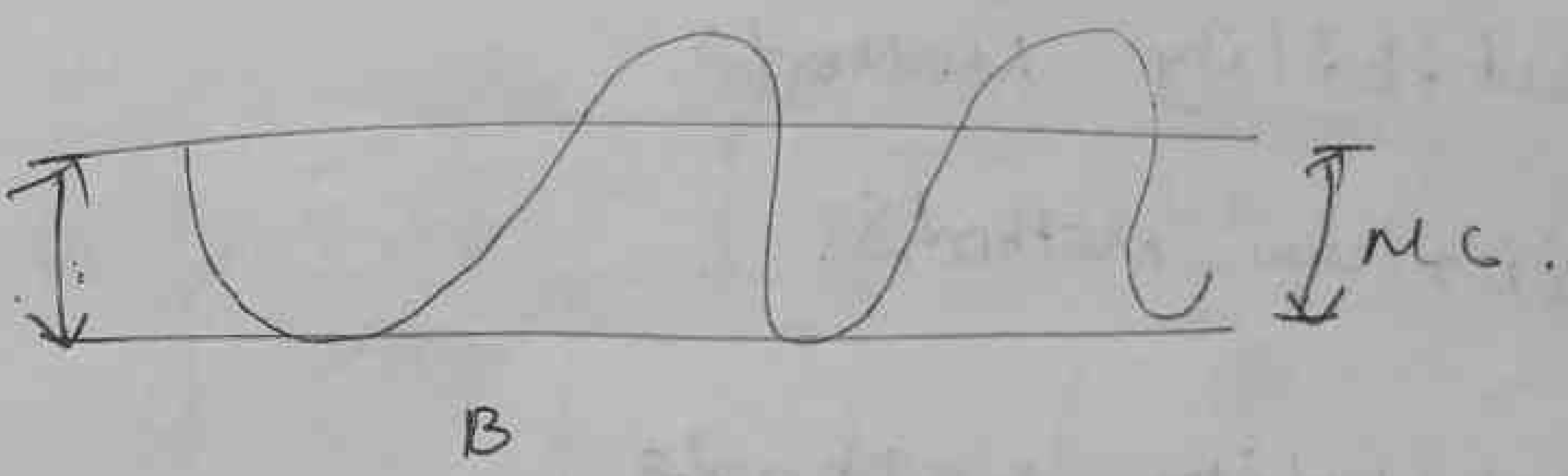
(1) slope deflection method.

(2) moment distribution method.

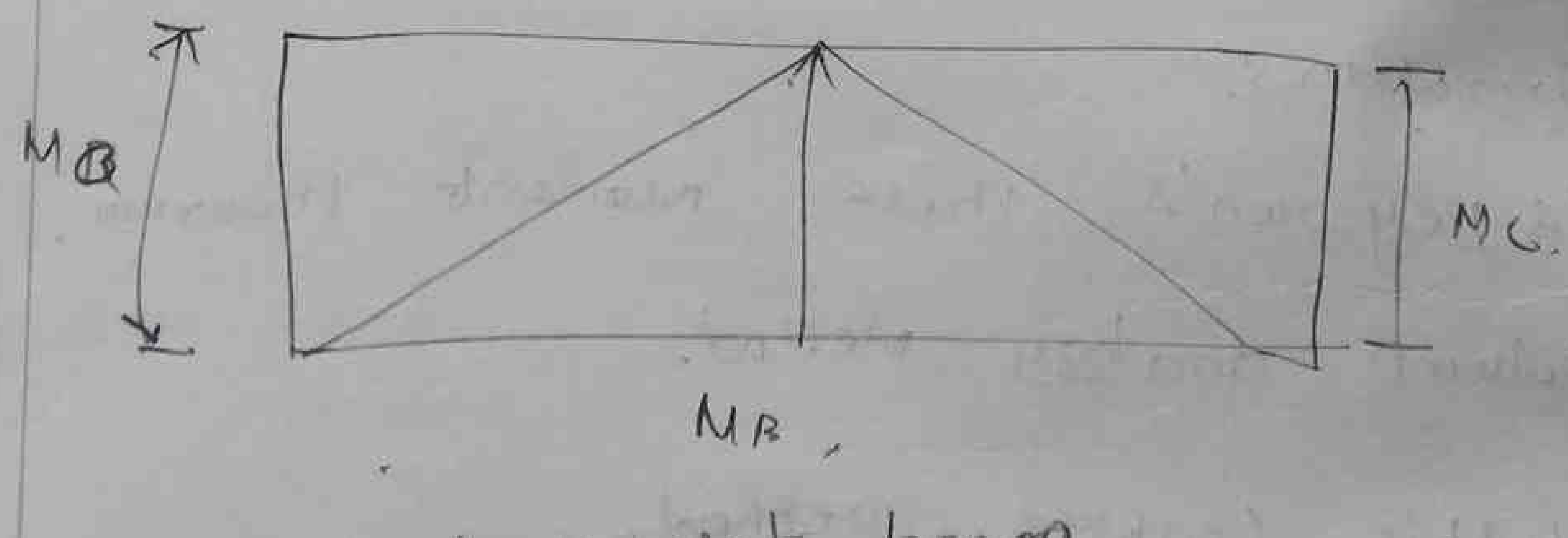
(3) minimum potential energy method.







Final BMD.



Support moment beam.

calpeyron's three moments theorem

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = \frac{6a_1 x_1}{l_1} + \frac{6a_2 x_2}{l_2}$$

1) problems Simply supported beam.
ABC is contunes over to is spans as above

Fixed BMD

Consider the span AB & BC as separate simply supported beams.

(a) Span AB

BM @ mid span $= \frac{w_1 l_1^2}{8}$

$= \frac{20\times 6^2}{8}$

$= 90$ kNM.

area, $a_1 = \frac{2}{3} \times 6 \times 90$

$= 360 \text{ kNm}^2$.

consider distance From A.

$x_{A1} = \frac{l}{2} = \frac{6}{2} = 3m$.

(b) span BC

Bm at D, $M_D = \frac{Wab}{l}$

$= 20$

1. AB Continuous beam ABCD is simply supported over free end spans of 6m, 5m and 4m, span AB is carry is a central point load of 80kN At the distance of 2m from B and the span CD carry is an uniformly distributed of 80kN per meter analysis beam the by ~~me~~ analyises clapyron,s theorem of three moment and draw SF and bending moment diagram.

Given data:

b = 5m

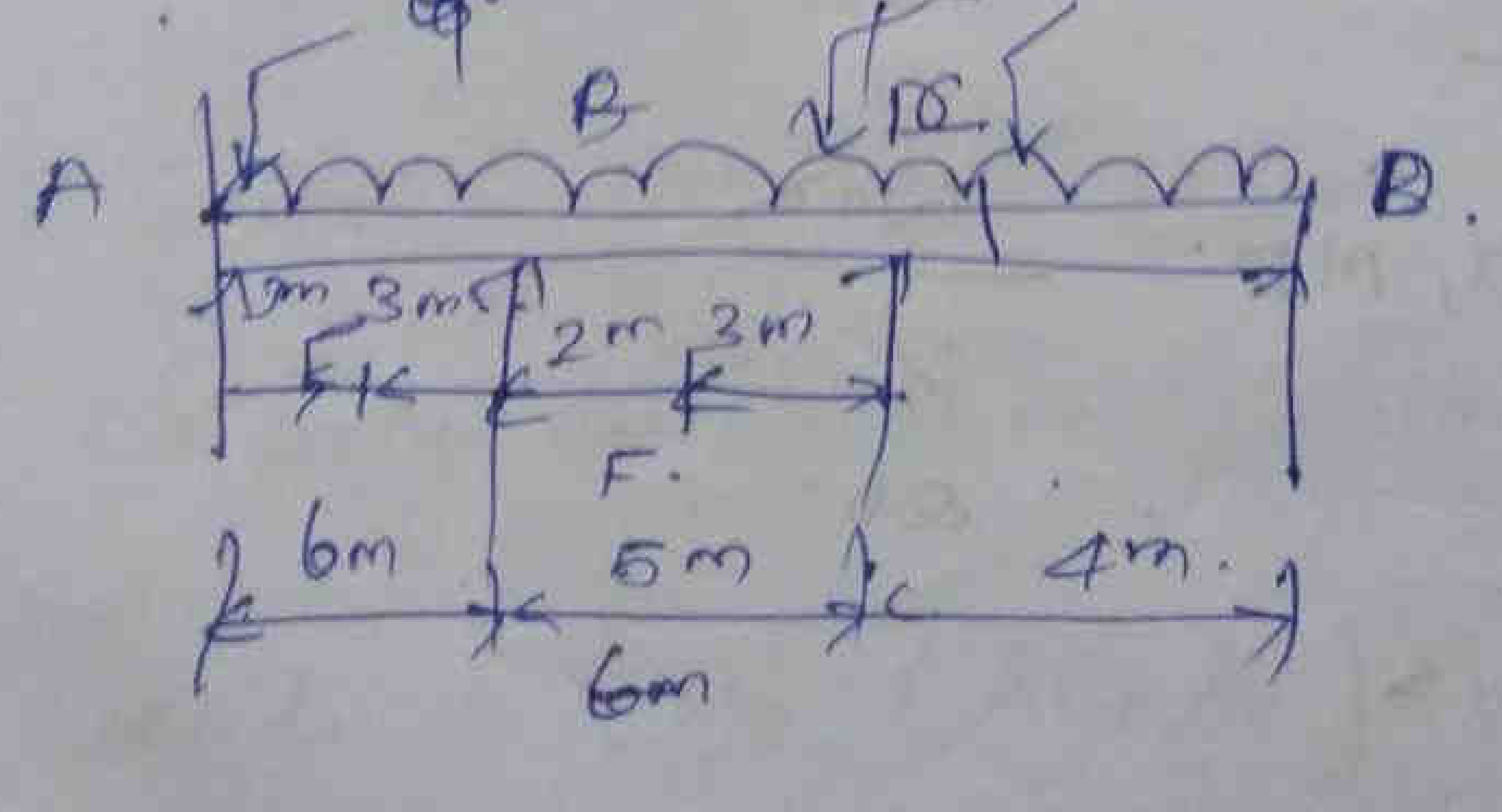
d = 4m

point load (w) = 80kN

udl (w) = 80kN.

span (l) = 6m.

Sol:



Span AB

BMD at E $= \frac{w_1 ab}{l_1} = \frac{90 \times 3 \times 3}{6}$

$= 135$ kNM.

$a_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \cancel{5 \times 4} \times 6 \times 135$

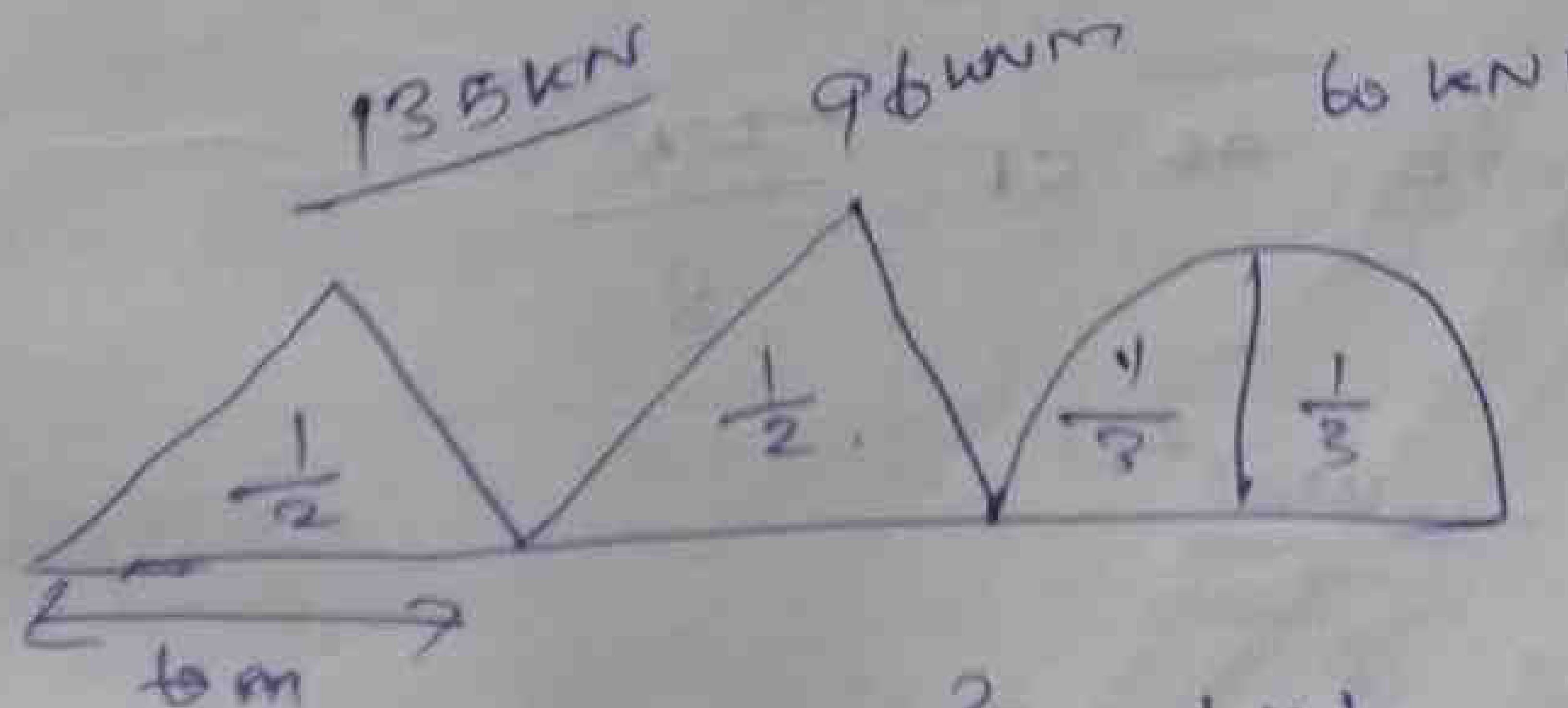
$= \cancel{10\ \text{kNm}^2}$.

$= 405\ \text{kNm}^2$

$\bar{x} a_1 = \frac{l_1 + a_2}{3}$

$= \frac{(6+3)}{3}$

$= \frac{9}{3} = 3$m.



$\frac{2}{3} \times b \times h$

Span B = $\frac{w_2 a b}{l_2} = \frac{80 \times 2 \times 3}{5}$

$F = 96$ kNm.

$a_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \cancel{40} \times 5 \times 96$

$= 240$ kNm^2.

$\bar{x} a_2 = \frac{(l_2 + a_2)}{3}$

$= \cancel{l_2 +}$

$= \frac{5+2}{3} = \frac{7}{3} = \cancel{5.6} \; 2.33$ m.

From (x)

$\bar{x} a_3 = \frac{l_2 + b}{3} = \frac{5+3}{3} = \frac{8}{3} = 2.6$ m

Span CD

bending moment at CD: $\frac{w_3 l^2}{8}$

$= \frac{w_3 l^2}{8}$

$= \frac{30 \times 4^2}{8} = \frac{120}{8} = \frac{48000}{8} = 60$ kN

From centroid C

$\bar{x}\,a_4 = \dfrac{\frac{l_4}{2}+c}{4}$

$= c$

centre of g:

$\bar{X} = \dfrac{l}{2} = \dfrac{4}{2} = 2$

$a_3 = \dfrac{1}{2} \times b \times h$

$a_3 = \dfrac{2}{3} \times b \times h$

$= \dfrac{2}{3} \times 4 \times 60$

$= 160\ kNm^2$

$\bar{x}\,a_4 =$

centroid from D

$\bar{x}\,a_4 = \dfrac{l_3}{2} = \dfrac{4}{2}\ \dfrac{4}{2} = 2m$

$M_A = 0 = M_D$.

Apply three moment equation

to span AB and BC.

$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \dfrac{6a_1 x_{a1}}{l_1} + \dfrac{6a_2 x_{a2}}{l_2}$.

$0 \times 6 + 2M_B \times (6+5) + M_C 5 = \left(\dfrac{6 \times 405}{6}\right) + \left(\dfrac{6 \times 2108}{5}\right)$

Scanned by TapScanner

$0 + 2M_B \times (11) + 5M_C = 1215 + 768$

$22 M_B + 5M_C = 1983 \quad \ldots \quad (1)$

$M_A = 0 = M_D$

to span BC and CD.

$M_B \times l_1 + 2M_C \,(l_1 + l_3) + M_B l_3 = \frac{6A_1 \bar{x}_{B_1}}{l_2} + \frac{6A_2 \bar{x}_2}{l_3}$

$M_B \times 5 + 2M_C\,(5+4) + 0 \times 4 = \left(\frac{6 \times 240 \times 2.33}{5}\right) + \left(\frac{6 \times 160 \times 2}{4}\right)$

$\boxed{5M_B + 18M_C = 1151} \rightarrow (2)$

(1) ~~$\times$ (2) $\times 22$~~

(1) $\times 5$ & (2) $\times 22$

$110M_B + 25M_C = 9915$

$110M_B + 396M_C = 25322$
(−) (−) (−)

$+371M_C = +15407$

$M_C = \frac{15407}{371}$

$M_C = 41.6$ kNm.

Scanned by TapScanner

$5M_B + 18M_C = 1151$

$5M_B + (18M_C \times 41.6) = 1151$

$5M_B + 748.8 = 1151$

$5M_B = 1151 - 748.8$

$5M_B = 402.2$

$M_B = \frac{402.2}{5}$

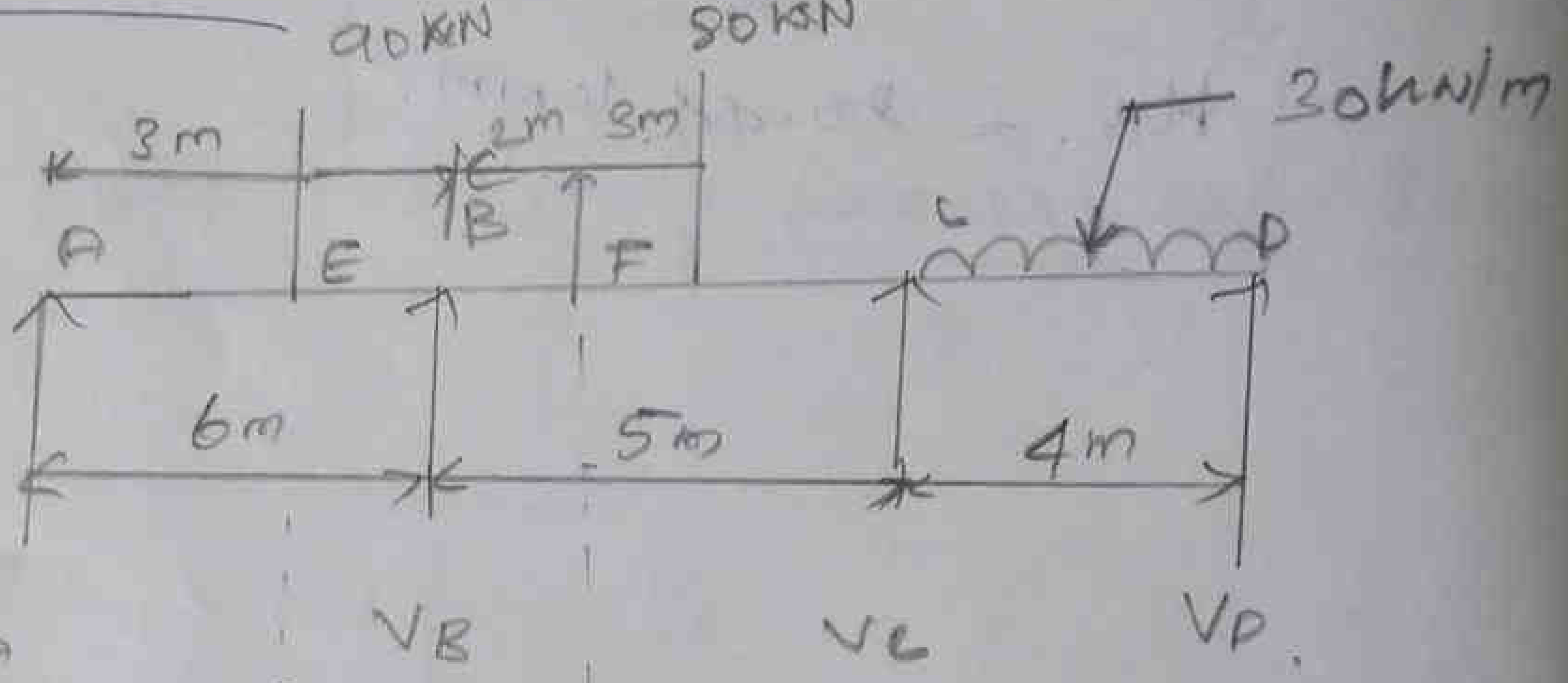
$\boxed{M_B = 80.44 \text{ kNm.}}$

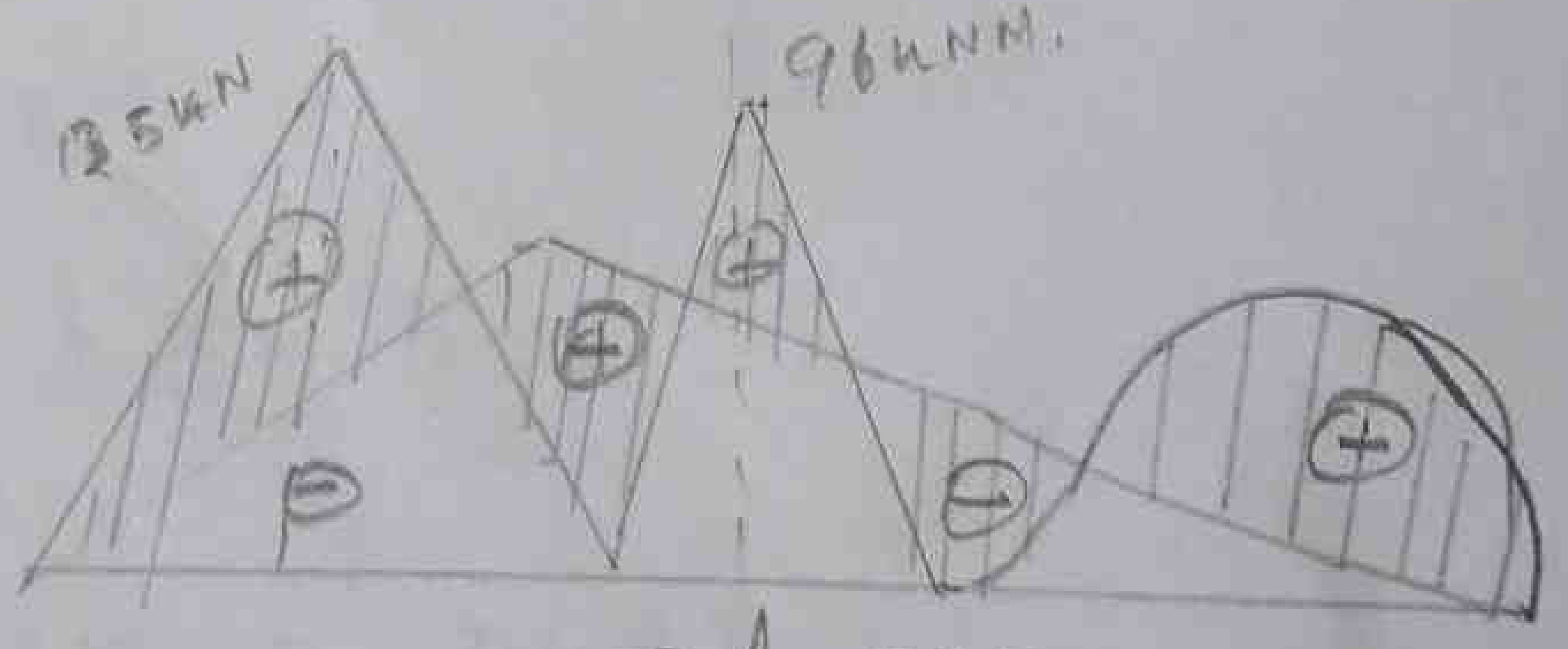
$V_A + V_B + V_C + V_D = 90 + 80 + (30 \times 4)$

$V_C = (90 + 80 + 120) - (V_A - V_B - V_D)$

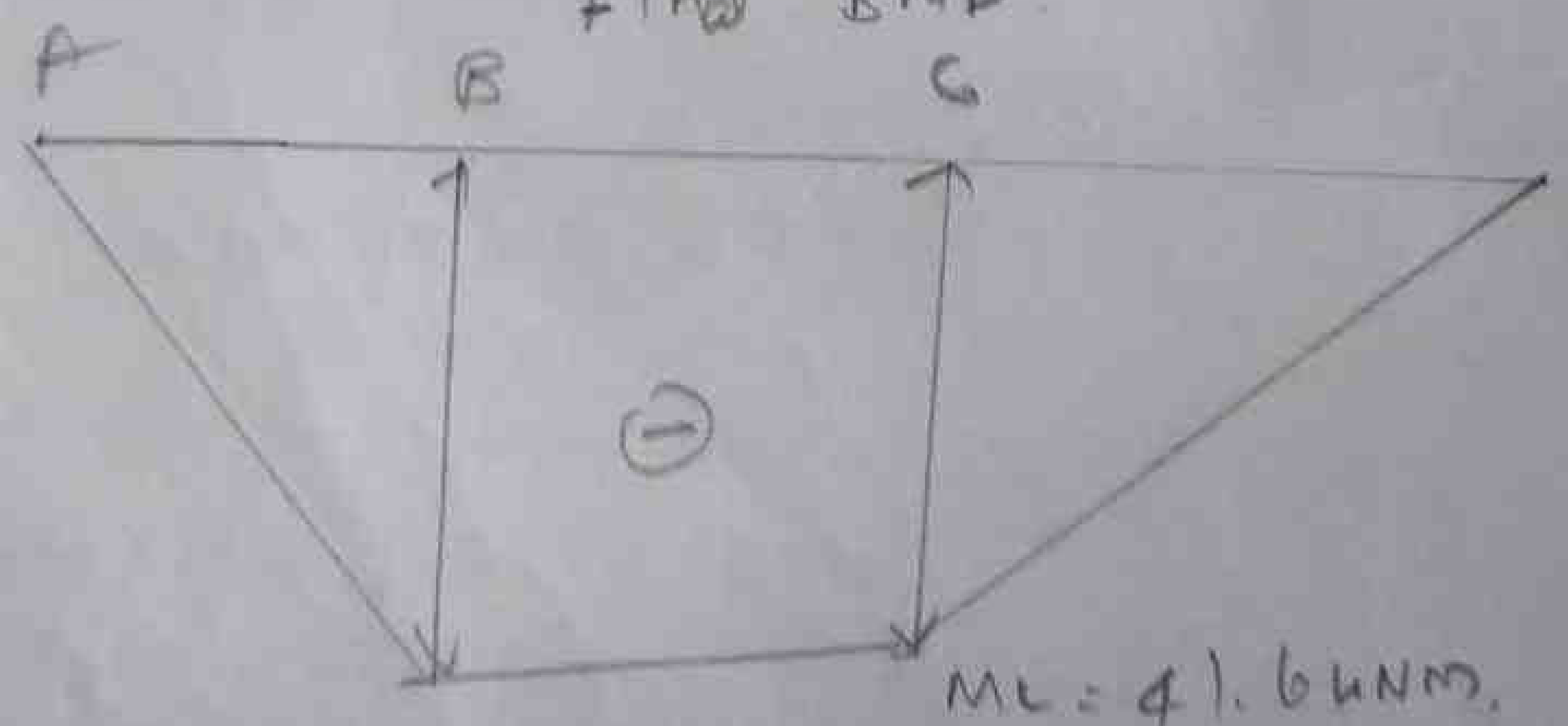
$= 290 - 31.6 - 174.11 - 49.6$

$V_C = 34.7\,kN.$



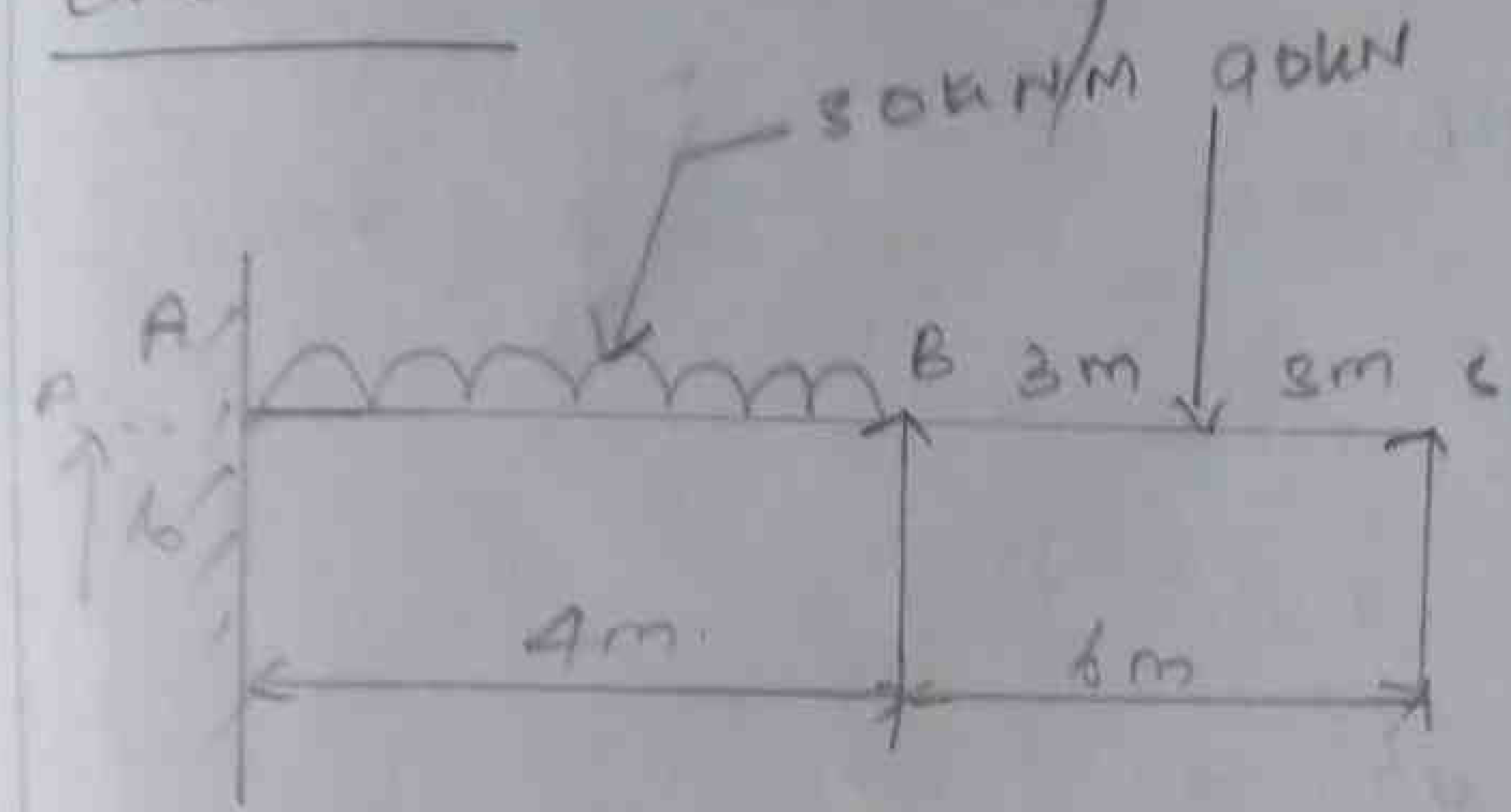


Find BMD



Analyse the continuous shown in figure by the use clapyron's theorem of three moment draw SFD and BM diagram.

Given:



span AB, $l_1 = 4m$, $w_1 = 30kN/m$.

span BC, $l_2 = 6m$, $w_2 = 90kN$ (central point).

Sol:

The beam is fixed at the end A. Hence assume an imaginary line zero span A_0A to the left of A with zero loading.

(i) Free BMD:

Consider the spans A_0A, AB & BC are separately simply supported beams.

(a) span A_0A.

It is imaginary span with zero loading...

Area of A_0A_1, $a_0 = 0$.

Centroid of A_0A_1, $x_A = 0$

BMD

span AB =

$$BM = \frac{wl^2}{8}$$

$$= \frac{30 \times 4^2}{8}$$

$$= 60\,kNm.$$

$$\text{Area } AB_1,\ a_1 = \frac{2}{3} \times b \times h$$

$$= \frac{2}{3} \times 4 \times 60$$

$$= 160\,kNm^2$$

$$\text{Centroid } x_{A_1} = \frac{l}{2}$$

$$= \frac{4}{2} = 2m$$

span BC

$$BM = \frac{wab}{l_2}$$

$$= \frac{90 \times 3 \times 3}{6} = 135\,kNm$$

Area of BBC, $a = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 135$

$= \underline{202.5 m^2}$

centroid $\bar{x} =$ $= \underline{405 kNm^2}$

$= \frac{6+b}{3}$

$= \frac{6+3}{3}$

$= \frac{9}{3} = \underline{3m}$

Support Moments :

let M_{A0}, M_A, M_B & M_C. are support hogging moments.

End support $M_{A0} = 0$.

End. support, $M_C = 0$.

$M_{A0} l_0 + 2M_A (l_0 + l_1) + M_B (l_2) = \frac{6a_0 \times \bar{x}_0}{l_0} + \frac{6A_1 \times \bar{x}_1}{l_1}$

$0 + 2M_A (0+4) + M_B \times 4 = \frac{6 \times 0 \times 0}{0} + \frac{6 \times 160 \times 2}{4}$

$\boxed{8M_A + 4M_B = 0 + 480 \rightarrow (1)}$

$$M_A l_1 + 2M_B(l_1+l_2) + M_C(l_2) = \frac{6a_1 \bar{x}_{A_1}}{l_1} + \frac{6A_2 \bar{x}_{A_2}}{l_2}$$

$$M_A \times 4 + 2M_B(4+6) + M_C(6) = \frac{6 \times 160 \times 2}{4} + \frac{6 \times 405 \times 3}{6}$$

~~$4M_A + 8M_B + 2\,6M$~~

$$4M_A + 20M_B + 0 = 480 + 1215$$

$$\boxed{4M_A + 20M_A = 1695} \rightarrow (2)$$

$$(1) \div (2) = 4M_A + 2M_B = 240$$

$$(2) \div 1 = 4M_A + 20M_B = 1695$$

(-) (-) (-)

$$-18M_B = -1455$$

$$M_B = \frac{1455}{18}$$

$$\boxed{M_B = 80.883 \text{ KNm}}$$

$$8M_A + 6M_B = 480$$

$$8M_A + (6 \times 80.88) = 480$$

~~484.98~~ 322.32

$8M_A =$ ~~514.98~~ $= 480$

$8M_A +$ ~~480~~ $\times$ ~~513.48~~ 484.98 $-$ ~~498~~ $= 0$

156.68 = 0

$$8M_A = \cancel{-33.48} \quad - \frac{\cancel{47.98}}{\cancel{8}} \quad \frac{156.68}{8}$$

$$M_A = \frac{\cancel{33.48}}{\cancel{8}} =$$

$\cancel{M_A = -4.186}$ ~~kNm~~ $M_A = \underline{19.585\,kNm}$

UNIT - 3

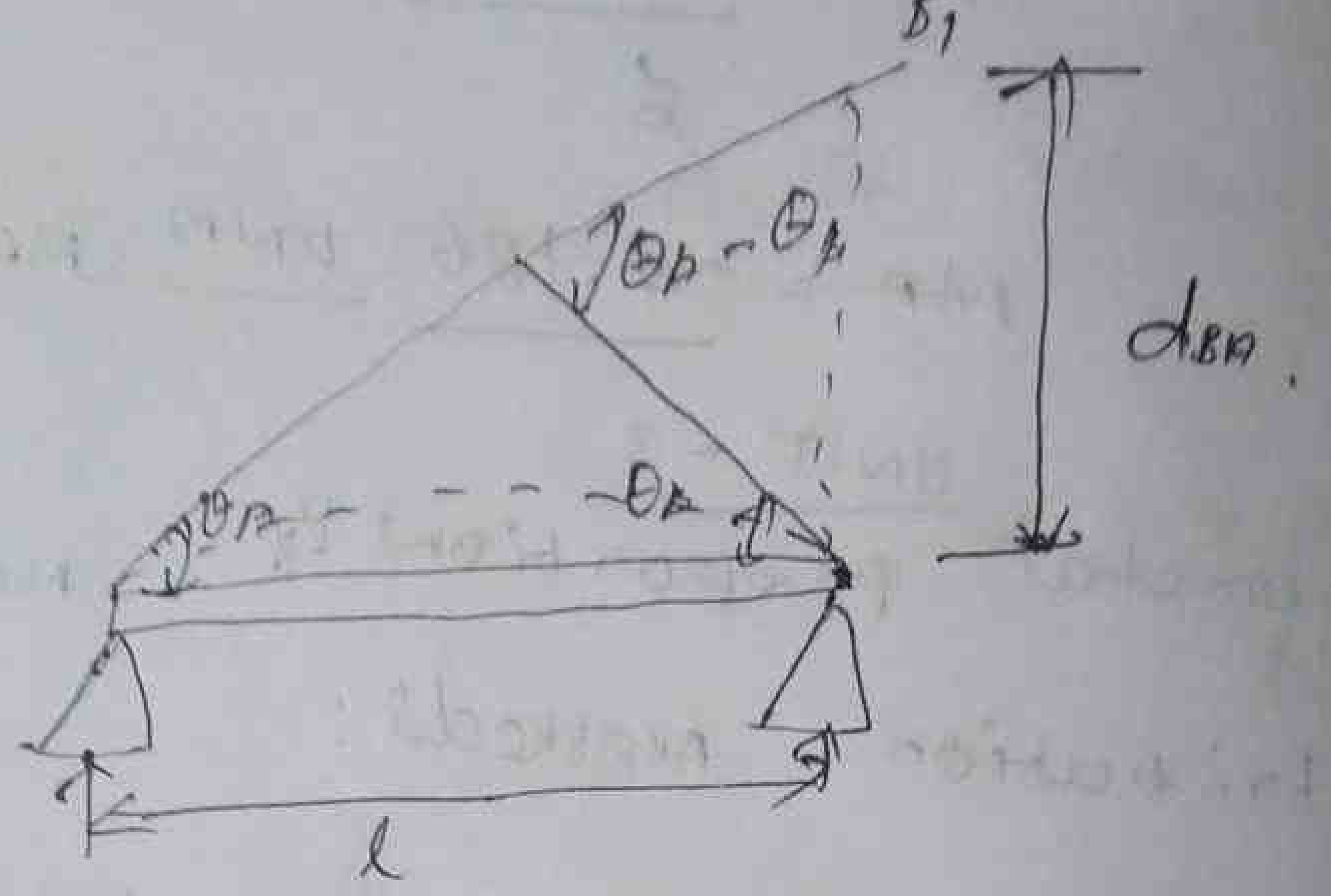
Fundametal proportions of moment distribution methods:

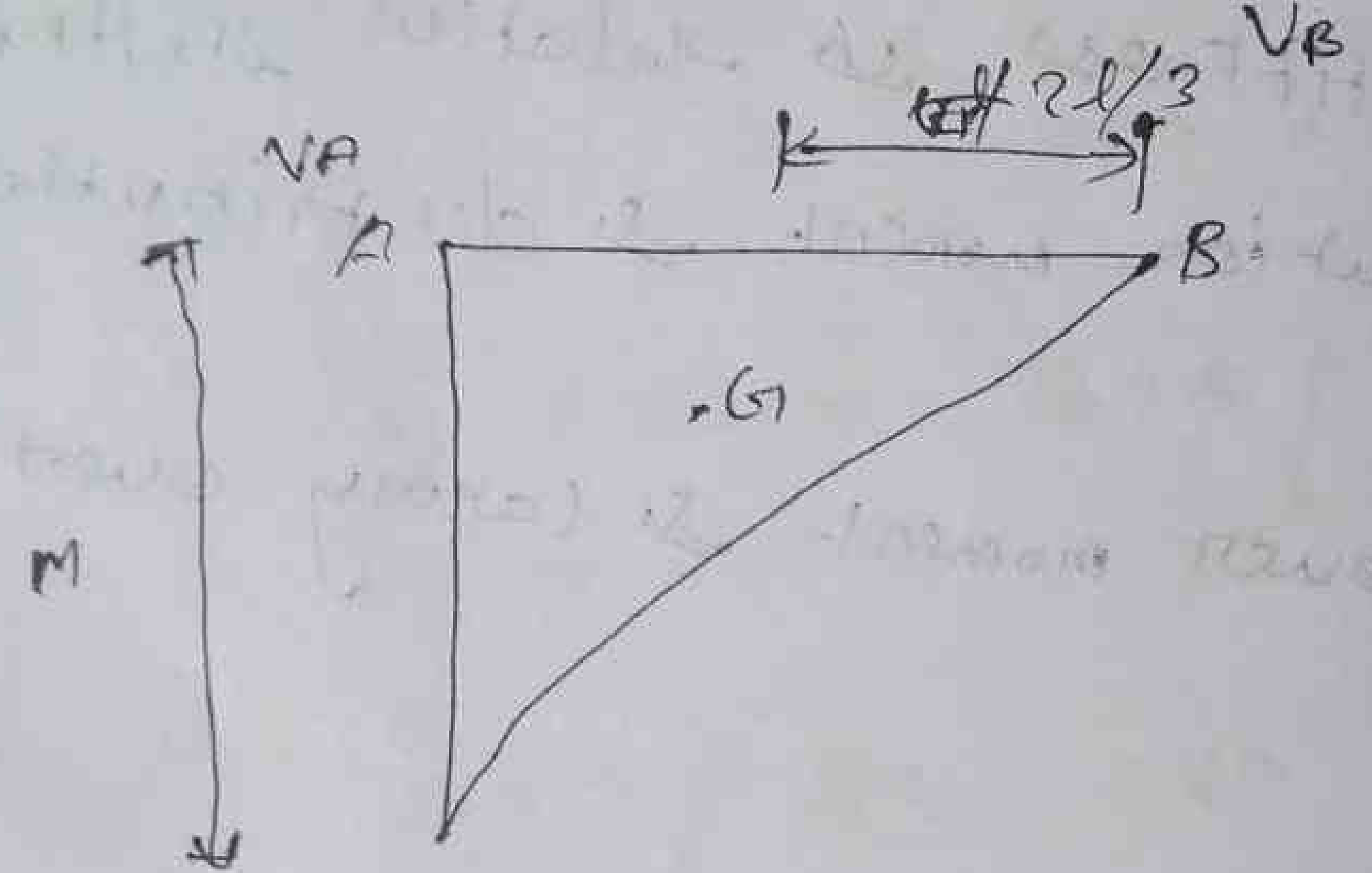
i) Beam stiffness & relative stiffness
ii) Distribution moment & distribution factor.
iii) carry over moment & carry over factor.

Explained by

1) Hinged at both ends.
2) Hinged at one end & fixed at other end.
3) several beam members meeting at a joint.

Beams Hinged at both ends.





Slope at A, $\theta_A = \frac{Ml}{3EI}$

Slope at B, $\theta_B = \frac{Ml}{6EI}$

Reaction :

$V_A = \frac{M}{l}$

$V_B = \frac{M}{l}$

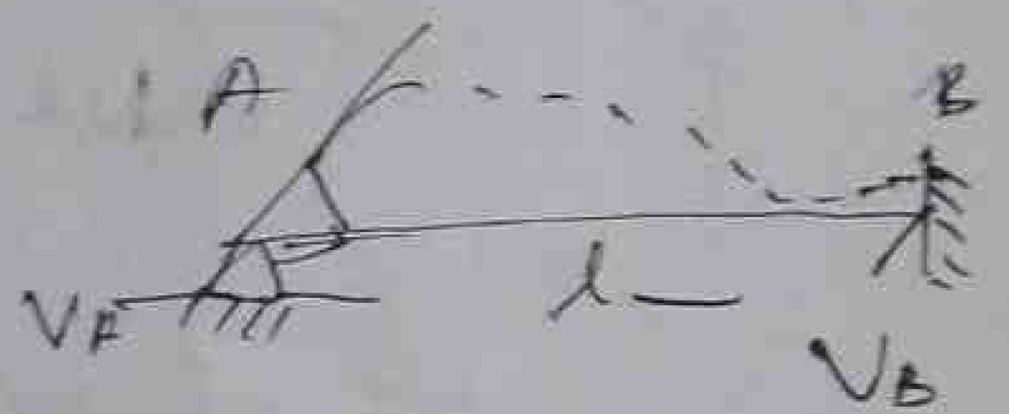
Stiffness. (or) absolute stiffness (or)

Flexural stiffness (k).

$$k = \frac{\text{Applied moment at A}}{\text{Slope at A.}}$$

$$k = \frac{3EI}{l}.$$

Hinged at one end and fixed at other end:



moment, $M_A = -M.$

moment, $M_B = -M + V_A l$ (or) $= -M + \frac{3M}{2l} \times l$

$$= \frac{2M}{2}$$

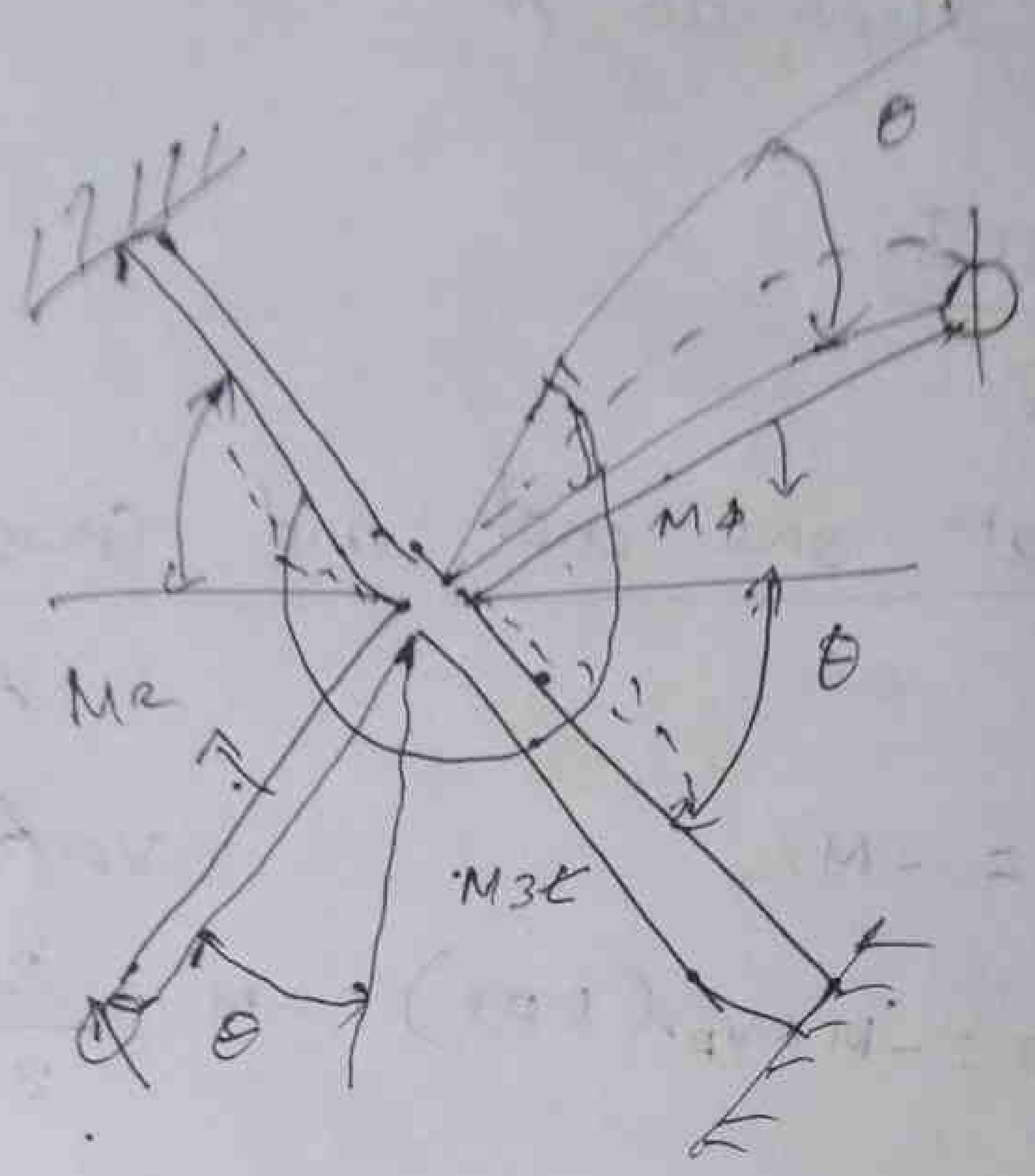
slope at A, $\theta_A = \frac{MI}{4EI}$

$$V_A = -\frac{3M}{2l}$$

$$V_B = \frac{3M}{2l}.$$

$$\text{Stiffness} = \frac{4EI}{l}.$$

Several beam members meeting at a joint



$$M = M_1 + M_2 + M_3 + M_4$$

$$M_1 = \frac{4E_1 I_1 \theta}{l_1} = k_1 \theta.$$

$$M_2 = \frac{3E_2 I_2}{l_2} \theta = k_2 \theta.$$ (since far end B is hinged)

$$M_3 = \frac{4E_3 I_3}{l_3} \theta = k_3 \theta.$$ (since far end C is fixed).

$$M_4 = \frac{3E_4 I_4}{l_4} \theta = k_4 \theta$$ (since far end D is hinged).

$$M_1 : M_2 : M_3 : M_4 = k_1 : k_2 : k_3 : k_4$$

$M_1 + M_2 + M_3 + M_4 = k_1\theta + k_2\theta + k_3\theta + k_4\theta$.

$$M = (\Sigma k)\theta.$$

$$\frac{M_1}{M} = \frac{k_1\theta}{(\Sigma k)\theta} = \frac{k_1}{\Sigma k} \text{ (or) } M_1 = \frac{k_1}{\Sigma k} M$$

$$\frac{M_2}{M} = \frac{k_2}{\Sigma k} \text{ (or) } M_2 = \frac{k_2}{\Sigma k} M.$$

$$\frac{M_3}{M} = \frac{k_3}{\Sigma k} \text{ (or) } M_3 = \frac{k_3}{\Sigma k} M.$$

$$\frac{M_4}{M} = \frac{k_4}{\Sigma k} \text{ (or) } M_4 = \frac{k_4}{\Sigma k} M.$$

$$\frac{k_1}{\Sigma k}, \frac{k_2}{\Sigma k}, \frac{k_3}{\Sigma k} \;\&\; \frac{k_4}{k}.$$

are unown as distribution factors and M_1, M_2, M_3 & M_4 are distribution moment (or) balan cing moment.

1) Stiffness $=$ $\dfrac{\text{Applied moment at end of beam}}{\text{slope at that end of the beam}}$

2) Relative Stiffness

Ratio of stiffness of various members meeting at a structured joint.

3) Distribution factor. (D.F)

$$D.F = \frac{k_2}{\Sigma k}.$$

4) Distributed moment:

moment shared by a member at a joint in the proportion of stiffness or in relation of distribution factored to restore equilibrium of the joint in a direction opposite to applied moment.

5) Carry over moment

moment produced at the far end of a prismatic beam by the rotation of free end due to applied moment.

6) Carry over factor.

$$\text{carry over factor} = \frac{\text{carry over moment}}{\text{applied moment.}}$$

position:/

Hinged at both ends, $k = \frac{3EI}{\ }$

Continuous Beams involved in Moment Distribution Method.

1) Calculate the stiffness & relative stiffness for all members.

2) Calculate Distribution factor (D.F) of members meeting at intermediate joints.

D.F for fixed end member is zero.

D.F for hinged end member is one.

D.F for over hanging end member is zero.

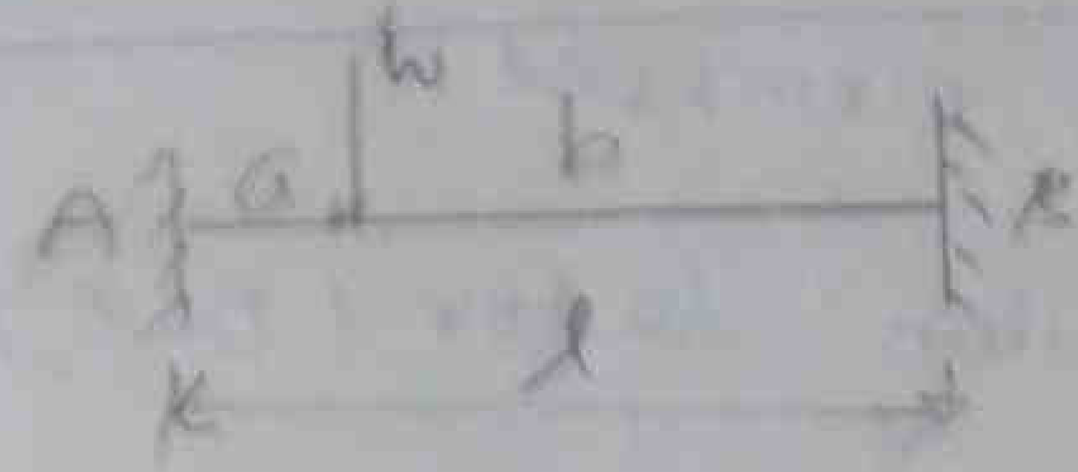
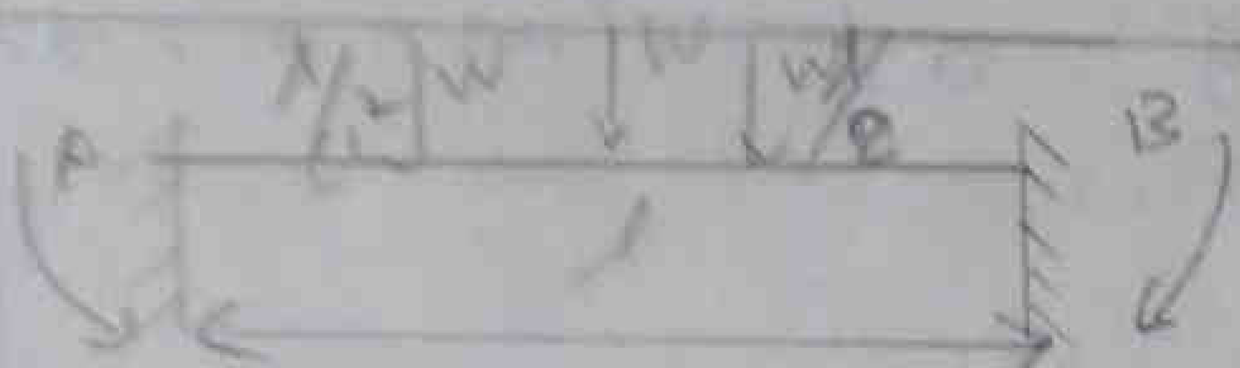
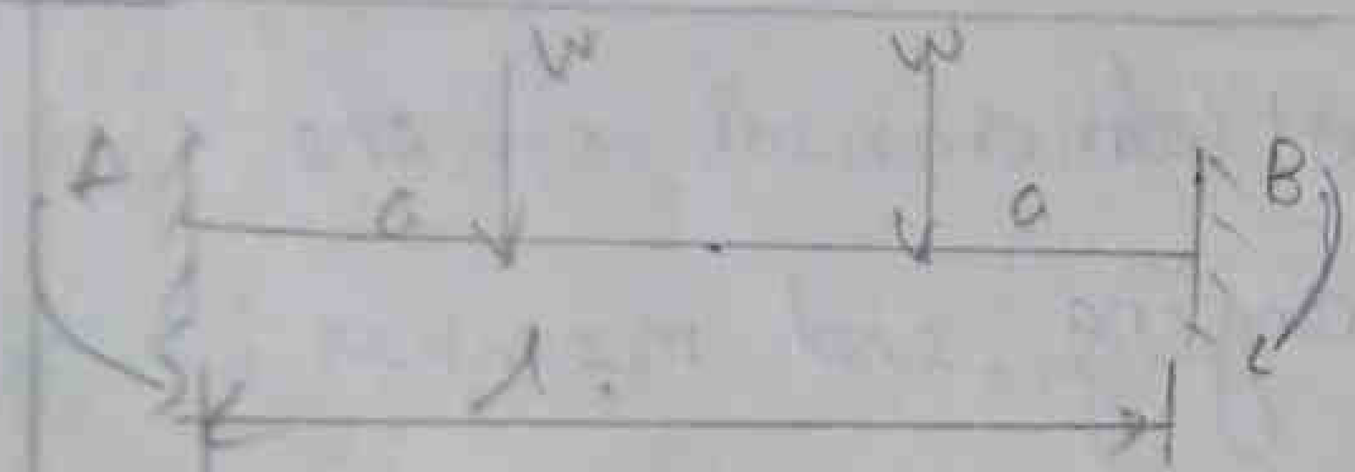
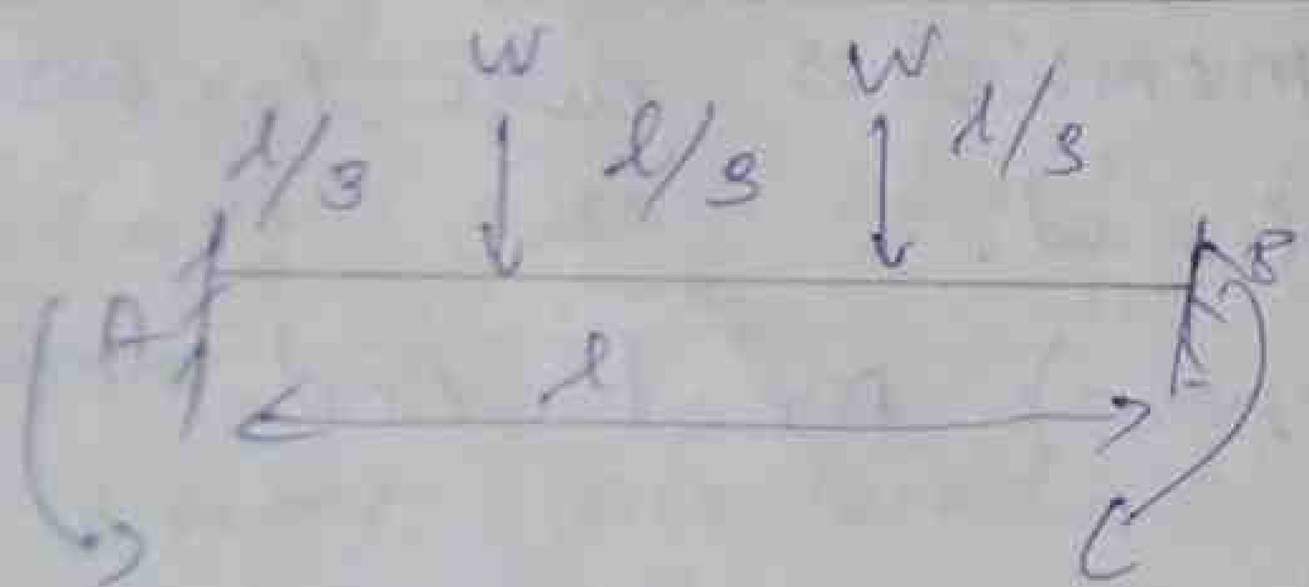
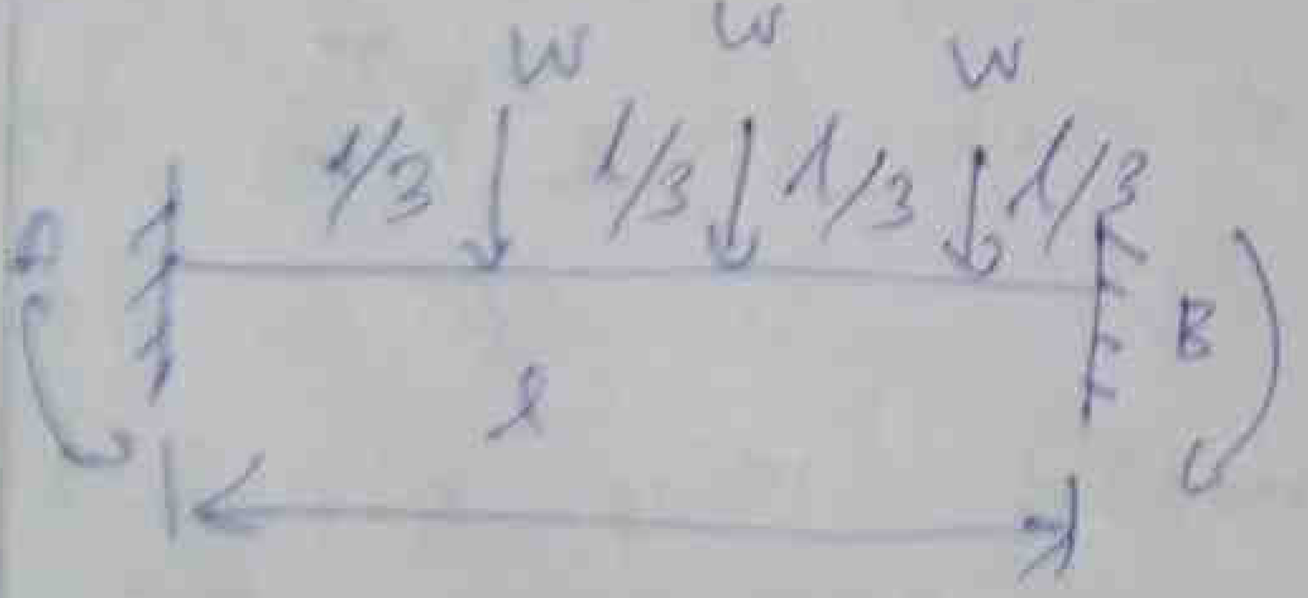
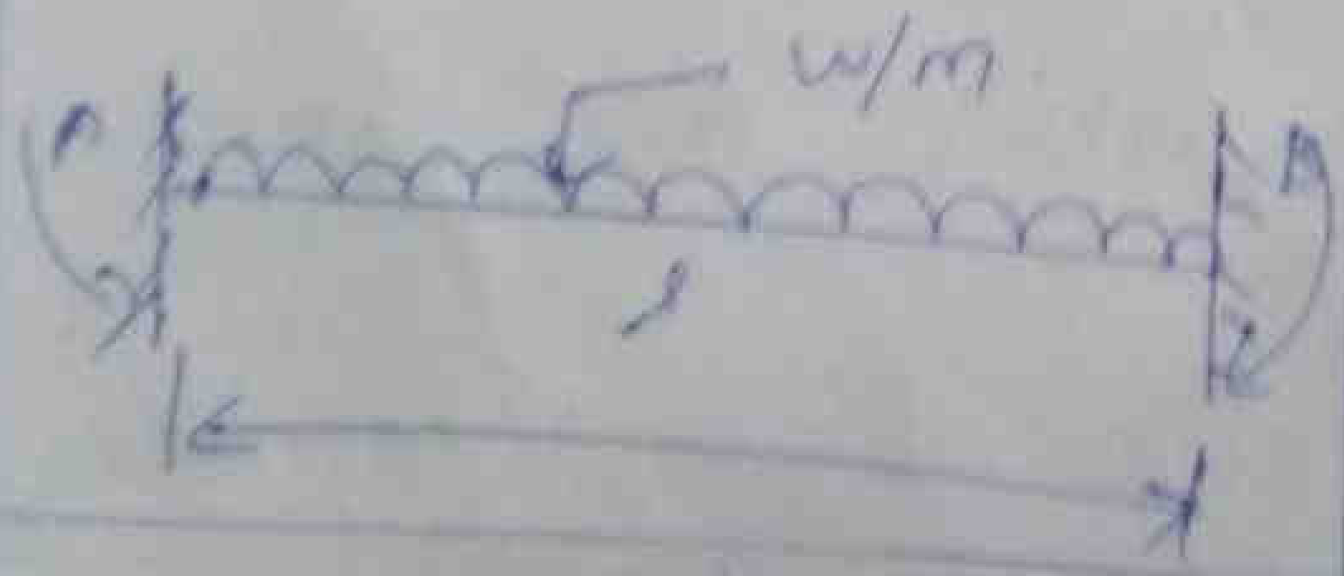
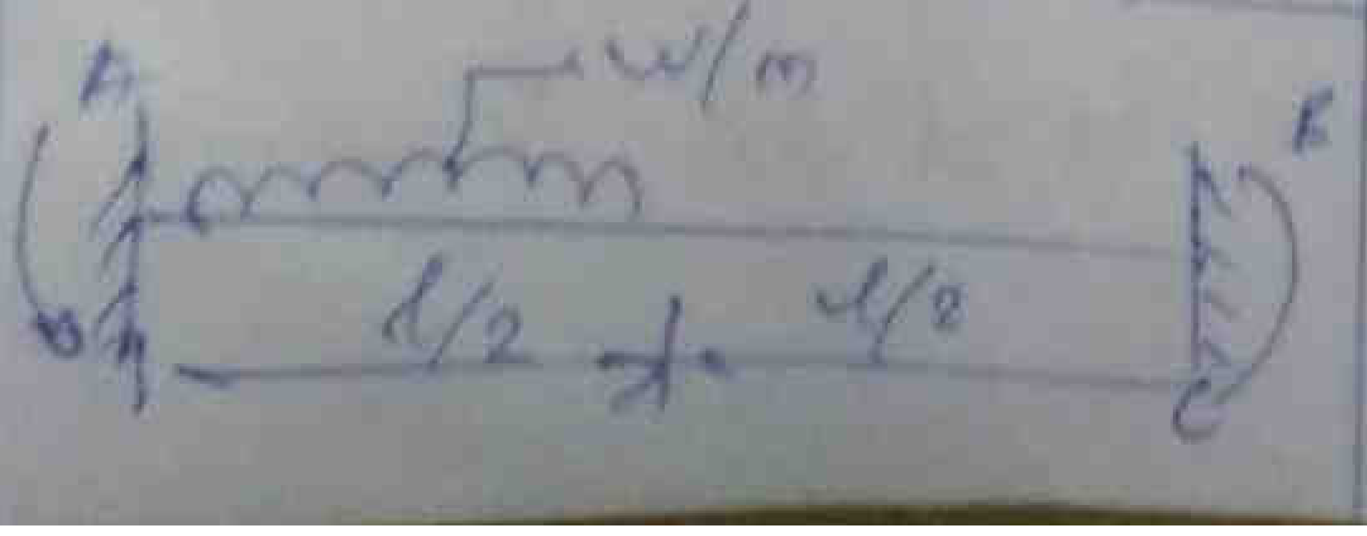
3) Assume all members are locked (or) clamped (or) fixed.

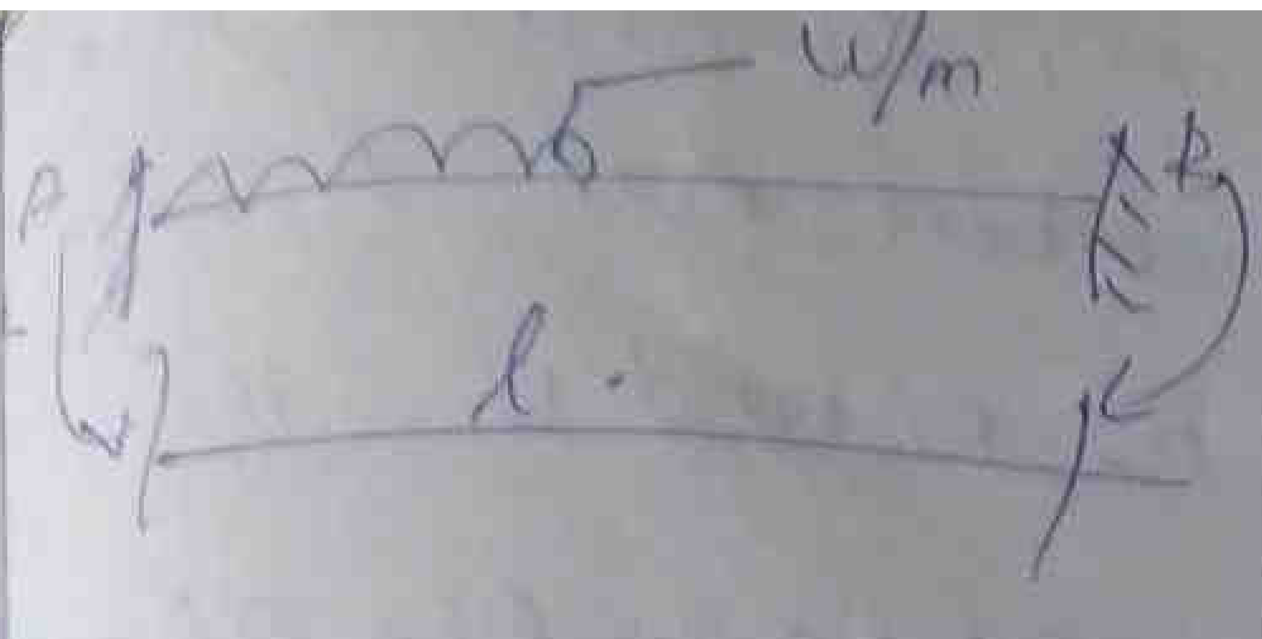
calculate the (FEM) fixed End moments.

stiffness fixed end. Stiffness hinged end

$$= \frac{4EI}{l} \qquad = \frac{3EI}{l}$$

Fixed end moment for standard case.

S.No	Fixed beam with load	Fixed end moment M_{AB}	Fixed end moment M_{BA}
1.		$(-)\frac{wab^2}{l^2}$	$(+)\frac{wa^2b}{l^2}$
2		$(-)\frac{wl}{8}$	$(+)\frac{wl}{8}$
3		$(-)\frac{wl(l-a)}{l}$	$(+)\frac{w(l-a)}{l}$
4.		$(-)\frac{2}{9}wl$	$(+)\frac{2}{9}wl$.
5		$(-)\frac{5}{16}wl$	$(+)\frac{5}{16}wl$
6		$(-)\frac{wl^2}{12}$	$(+)\frac{wl^2}{12}$
7		$(-)\frac{11}{192}wl^2$	$(+)\frac{15}{192}wl^2$

	$(-)\dfrac{w^2a}{12l^2}(6l^2-8al+3a^2)$	$(+)\dfrac{wa^2}{12l^2}(4l-3a)$

Application of Distribution method.

It is a ~~fa~~ practical approach method which yield quick solution to Statically indeterminate beams and frames.

1) Several members and beams well-at rigid joint.
2) Continuous beam and fixed beam.
3) Continuous beam extend simply supported ends.
4) Continuous with overhanging beams.
5) propped cantilever beam.
6) Symmetrical portal frames

Steps.

1. Stiffness of member $\Rightarrow \dfrac{4EI}{l}$ (fixed ends)
2. relative stiffness unit $=(k)$ $\dfrac{3EI}{l}$ (hinged ends)
3. Distributed factor (D.F) $= \dfrac{k}{\Sigma k}$ $\times$ 6 moment
4) Distributed moment.
5. Carry over moment. (co).

1) Five members OA, OB, OC, OD and OE meet rigid joint O. the member are A and OC and Fixed at B, E, and D. the length of the member, OA, OB, OC, OD and OE are length 3m, 4m, 3.6m, 4.5m and 5m. and there moments of interia are $4 \times 10^8 mm^4$, $3 \times 10^8 mm^4$, ~~2x4~~ $2.4 \times 10^8 mm^4$, $2.7 \times 10^8 mm^4$ and $4.5 \times 10^8 mm^4$ respectively. IF a clockwise moment of 60kNm is applied at the joint O, Calculate.

1) Distribution factor and(or) balancing distribution moment shared by the members.

2) Carry over moment farther ends.

Given data:

5 members joint at O.

length		I	
hinged → OA = 3m		OA	= $4 \times 10^8 mm^4$
hinged → OB = 4m.		OB	= $3 \times 10^8 mm^4$
Fixed / hinged → OC = 3.6m		OC	= $2.4 \times 10^8 mm^4$
Fixed → OD = 4.5m		OD	= $2.7 \times 10^8 mm^4$
Fixed → OE = 5m.		OE	= $4.5 \times 10^8 mm^4$

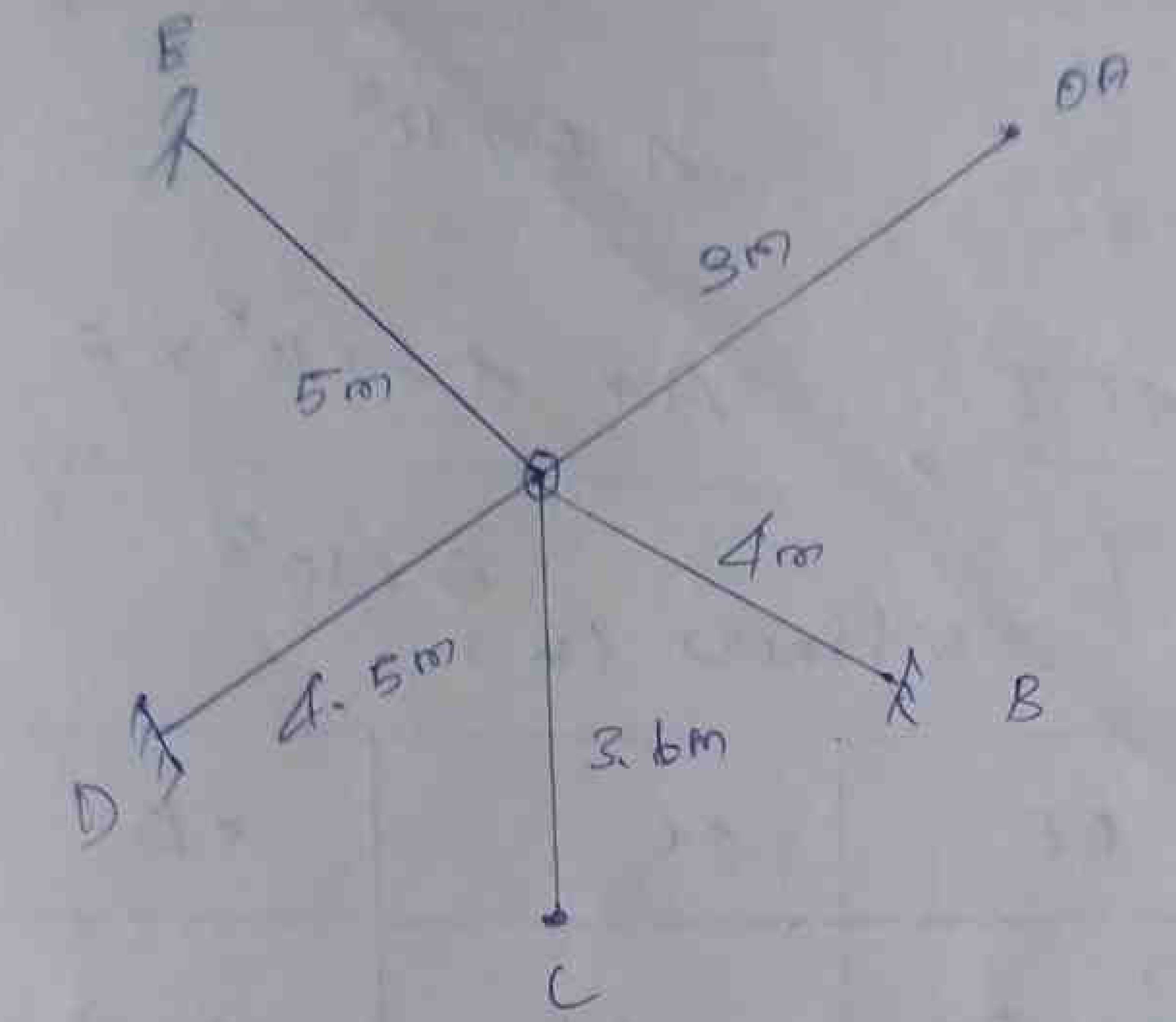
moment of Interia

Clockwise moment at O = +60kNm.

find

i) Distributution factor, Distribution moment and carry over moment further ends

sol:



~~Stiffness~~

$$\cancel{O A} = \frac{\cancel{3} EI}{\cancel{I} l} = \frac{\cancel{3 \times 4 \times 10^8 \times E}}{\cancel{3}} = \cancel{4 \times 10^8 E}$$

$$\cancel{OB} = \frac{\cancel{4EI}}{\cancel{I} l} = \frac{4 \cancel{\times 2.4 \times 10^8 \times E}}{4} = 2.\cancel{4 \times 10^8} E$$

$$\cancel{OC} = \frac{3EI}{l} = \cancel{3 \times}$$

$$\cancel{OB} = \frac{4EI}{l} = \frac{4 \times 3 \times 10^8 \times E}{4} : 3 \cancel{\times 10^8} E.$$

1) Stiffness, k

$$OA = \frac{3EI}{l} = \frac{3 \times 4 \times 10^8 \times E}{3 \times 10^8} E = 4 \times 10^5$$

$$OB = \frac{4EI}{l} = \frac{4 \times 3 \times 10^8 \times E}{4 \times 10^8} E = 3 \times 10^5$$

$$OC = \frac{3EI}{l} = \frac{3 \times 2.4 \times 10^8 \times E}{3.6 \times 10^3} \quad E = 2 \times 10^5 E$$

$$OD = \frac{4EI}{l} = \frac{4 \times 2.7 \times 10^8 \times E}{4.5 \times 10^3} \quad E = 2.4 \times 10^5 E.$$

$$OE = \frac{4EI}{l} = \frac{4 \times 4.5 \times 10^8 \times E}{5 \times 10^3} \quad E = 3.6 \times 10^5$$

2) Relative Stiffness (k).

OA	OB	OC	OD	OE.
$4 \times 10^5 E$	$3 \times 10^5 E$	$2 \times 10^5 E$	$2.4 \times 10^5 E$	$3.6 \times 10^5 E.$
4	3	2	2.4	3.6

Sum of Stiffness

$$\Sigma k = 4 + 3 + 2 + 2.4 + 3.6$$
$$= 15.$$

3) Distribution factor.

$$D.F @ OA = \frac{k}{\Sigma k} = \frac{4}{15}$$

$$D.F @ OB = \frac{k}{\Sigma k} = \frac{3}{15}$$

$$D.F @ OC = \frac{k}{\Sigma k} = \frac{2}{15}$$

$$D.F @ OD = \frac{k}{\Sigma k} = \frac{2.4}{15}$$

$$D.F @ OE = \frac{k}{\Sigma k} = \frac{3.6}{15}$$

4) Distribution moment

Apply moment at O = +60 kNm. (clockwise)

Applied moment in opposite direction must be applied.

Balancing (or) Distribution moment } = +60 kNm = -60 kNm.

D.M at OA = $\frac{-60 \times 4}{15}$ = -16 kNm.

D.M at OB = $-60 \times \frac{3}{15}$ = -12 kNm.

D.M at OC = $-60 \times \frac{2}{15}$ = -8 kNm.

D.M at OD = $-60 \times \frac{2.4}{15}$ = -9.6 kNm.

D.M at OE = $-60 \times \frac{3.6}{15}$ = -14.4 kNm.

5) Carry over moment (CO) for farther end.

CO to end A = 0. [Hinged end].

CO to end B = $\frac{1}{2} \times -12$ = -6 kNm.

CO to end C = 0. [Hinged end].

CO to end D = $\frac{1}{2} \times -9.6$ = -4.8 kNm.

CO to end E = $\frac{1}{2} \times -14.4$ = -7.2 kNm.

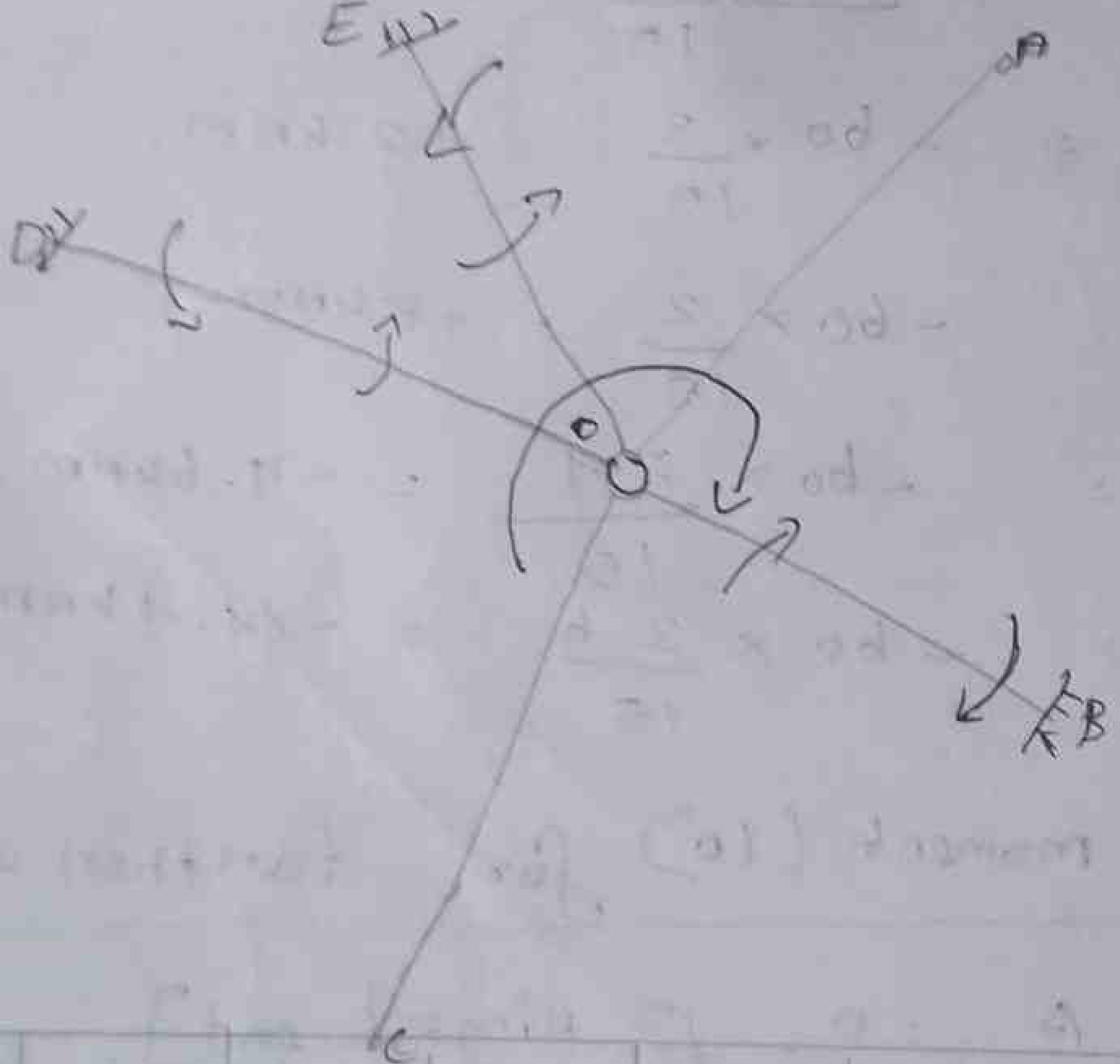
6) Applied moment for = -60 kNm.

* The direction of the distributed & carry over moment show in Fig.

* The process of Calculation & result are shown in tabulation

* Applying moment = +60 kNm.

* Applied moment. = −60 kNm.



Joint	member	span (mm)	E	I (mm⁴)	Stiffness (k)	$D.F = \frac{k}{\Sigma k}$	(distributed ~~carry over~~) Moment (kNm)
O	OA	3×10^3	E	4×10^8	$\frac{3 \times E \times 4 \times 10^8}{3 \times 10^5} = 4$	$\frac{4}{15}$	−16
O	OB	4×10^5	E	3×10^8	$\frac{4 \times E \times 3 \times 10^8}{4 \times 10^8} = 3$	$\frac{3}{15}$	−12
O	OC	3.6×10^3	E	2.4×10^8	2	$\frac{2}{15}$	−8
O	OD	4.5×10^3	E	2.7×10^8	2.4	$\frac{2.4}{15}$	−9.6
O	OE	5×10^3	E	1.5×10^8	3.6	$\frac{3.6}{15}$	−14.4

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carry over moment (kNm).

0 to A

-6 to B

0 to C

-4.8 to D

-7.2 to E

Strength of the hollow circular column $\Big\}$ $Ph = \frac{\pi^2 EI}{l^2}$

Interria $I = \frac{\pi(D^4 - d^4)}{64}$

$= \frac{\pi(100^4 - 80^4)}{64}$

$I = \underline{2.893 \times 10^6 \ mm^4}$

$Ph = \frac{\pi^2 EI}{l^2}$

$= \frac{\pi^2 \times 2\times10^5 \times 2.893\times10^6}{6000^2}$

$= 158.626\times10^5 N$

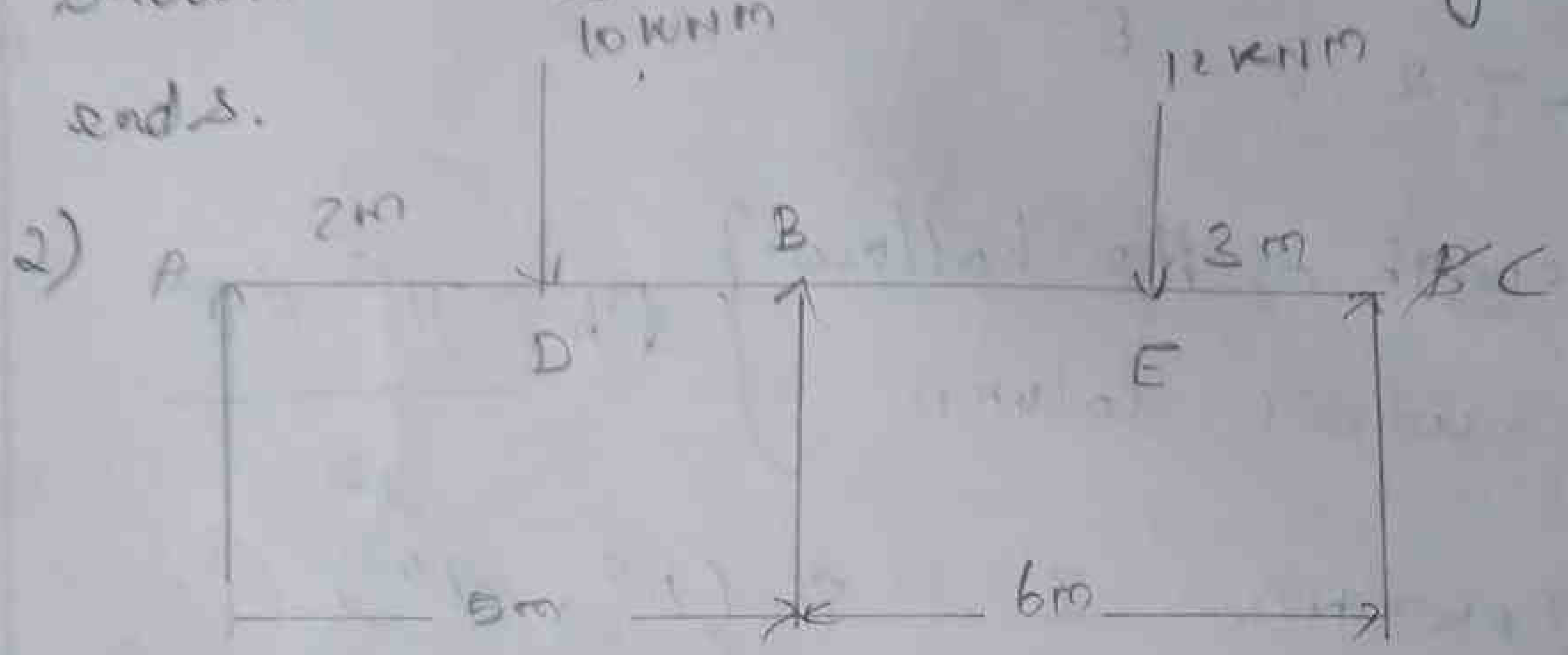
Ratio of the strength $= \frac{Ph}{Ps} = \frac{158.626\times10^5}{34.87\times10^5}$

$\boxed{\frac{Ph}{Ps} = 4.54}$

1] Continuous beam with simply support end.

1] Calculate the support moments by moment distribution method. and plot the bending moment and shear force diagram for the continuous beam. shown in figure. with simply supported ends.

2)



To find:

* support moment by moment distribution method.

* plot SF & B.M

Soln:

Distribution factor (D.F).

D.F AB = 1 [For end A is simply supported]

D.F BC = 1 [For end C is simply supported].

Joint B

stiffness of BA = $\frac{3EI}{l} = \frac{3EI}{5}$

stiffness of BC = $\frac{3EI}{l} = \frac{3EI}{6}$

Relative Stiffness (k).

Joint	Members	Stiffness	Relative Stiffness	Σk	D.F	Remark
BA	BA	$\frac{3EI}{5}$	6	$\frac{6}{11}$ 11	$\frac{6}{11}$	Multiply stiffness by $\frac{5\times6}{EI}$
	BC	$\frac{3EI}{6}$	5	$\frac{5}{11}$ 11	$\frac{5}{11}$	Relative (k).

$\Sigma k = 6 + 5$

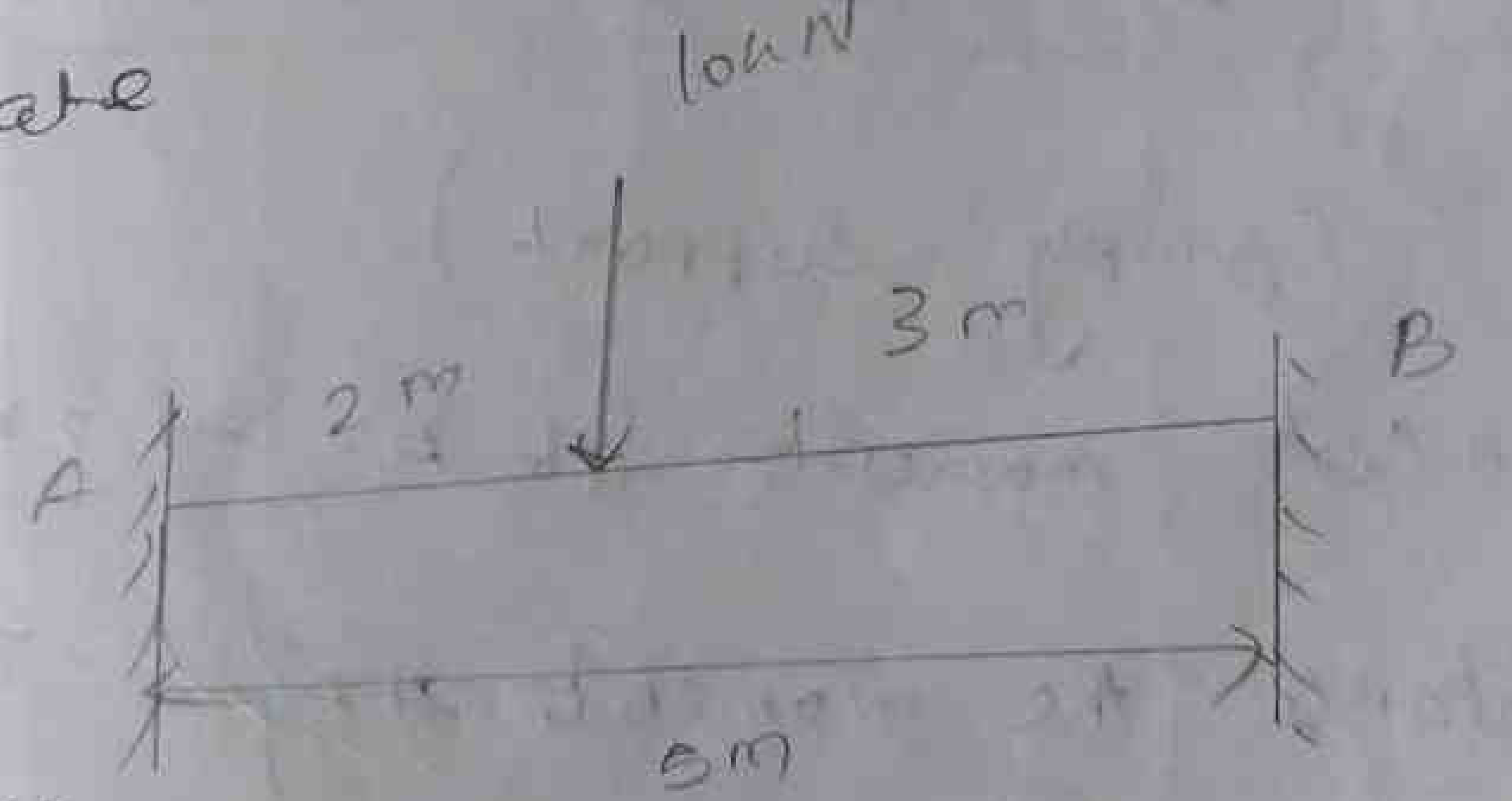
$= \underline{11}$

$D.F_{BA} = \frac{k}{\Sigma k} = \frac{6}{11} =$

$D.F_{BC} = \frac{k}{\Sigma k} = \frac{5}{11}$

Fixed End Moment (FEM).

Consider all support clocked. & Separate



Span AB

$M_{AB} = \frac{wab^2}{l^2} = \frac{10 \times 2 \times (3)^2}{(5)^2} = \cancel{36kNm}$ (-)7.2kNm

$M_{BA} = \frac{wa^2b}{l^2} = \frac{10 \times (2)^2 \times 3}{(5)^2}$ (+)4.8kNm

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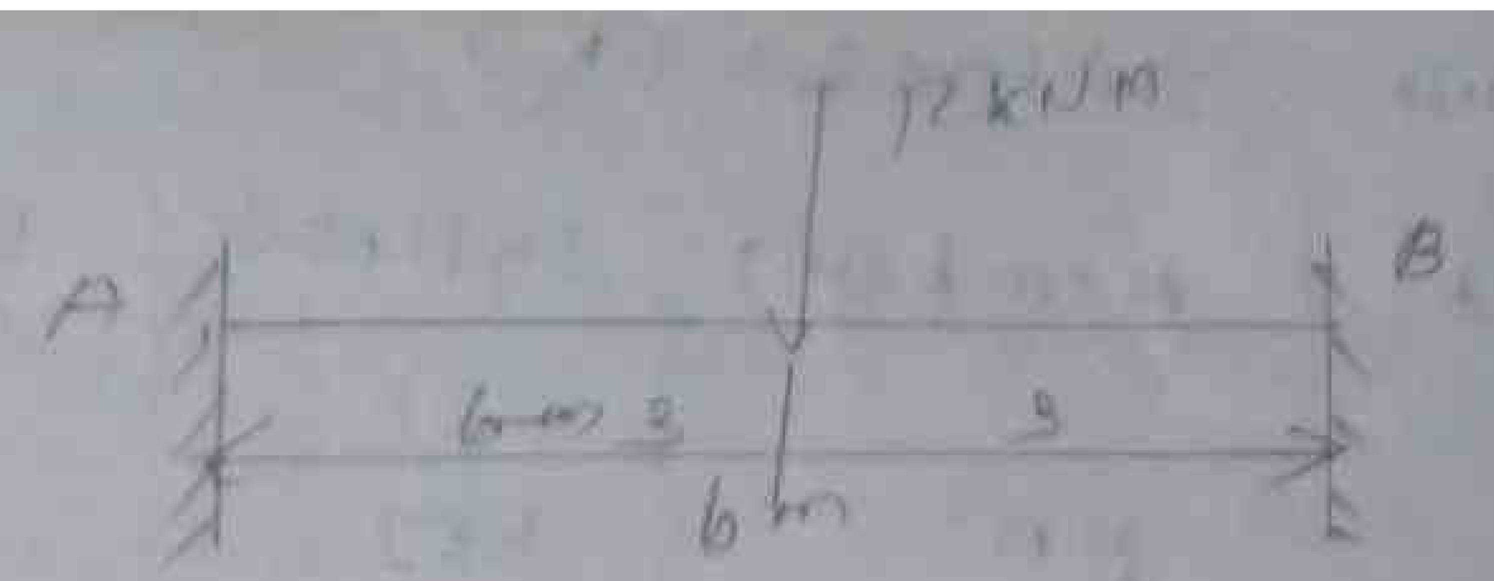
Span BC :



$$M_{BC} = \frac{wl}{8} = -\frac{12 \times 6}{8} = \underline{-9 kNm}$$

$$M_{CB} = \frac{wl}{8} = (+)\frac{12 \times 6}{8} = \underline{+9 kNm}$$

Moment distribution process & Table.

Enter D.F and FEM in the table.

i) Distribution of unbalanced moment

Release all the clamps and balanced the joint in a succession.

Distribute the unbalanced section

Joint A (simply support)

unbalanced moment at A = -7.2 kNm.

Joint balance the moment at A (-7.2 kNm)

= +7.2 kNm.

Joint C (s.s)

unbalanced

moment at C = +9 kNm.

Balanced moment at C = -9 kNm

= -9 kNm.

Joint B

unbalanced moment at B = + 4.8 kNm.

balanced moment at B = + 4.8 − 9

= $\underline{-4.2 \text{ kNm}}$

Balanced moment at B = − (−4.2 kNm)

= $\underline{+4.2 \text{ kNm}}$

Distribution to BA & BC

$BA = +4.2 \times \frac{6}{11} = \underline{2.29 \text{ kNm}}$

$BC = +4.2 \times \frac{5}{11} = \underline{1.90 \text{ kNm}}$.

carry over moment for end

co to from A to B = $+7.2 \times \frac{1}{2}$

= $\underline{3.6 \text{ kNm}}$.

co to from C to B : $\cancel{+4.2}$ −9 $\times \frac{1}{2}$

= $\underline{-4.5 \text{ kNm}}$

~~co to~~ fr.

This complete 1st cycle moment distribution.

Enter in 3rd step.

II cycle

unbalanced moment at B = 3.6 - 4.5
= 0.9 kNm.

Balancing moment = -(-9kNm) = 0.9 kNm

Redistribution moment to BA & BC again and enter in step 4.

Distribution to BA = $0.9 \times \frac{6}{11}$ = 0.490 kNm

Distribution to BC = $0.9 \times \frac{5}{11}$ = 0.41 kNm

Cycle	Joint	A	B		C
	Member	AB	BA	BC	CB
	D.F	1	5/11	5/11	1
I	1) FEM	-7.2	+4.8	-9.00	+9.00
	2) Release Distribution	+7.2	+2.29	+1.91	-9.00
II	3. Carry over	0.00	+3.6	-4.5	0.00
	4. Distribution	-	+0.49	+0.41	-
III	Co Final mom	-			-
	Distribute	-	~~+11.18~~		-
	Final moment	0.00	+11.18	-11.18	0.00
		-0.00 MA	-11.18	-11.8	MC 0
			MB Hogging		

4. Final Support Moments

If algebric sum to joint (+) to left & (-) ve to right is hogging in nature.

(-) to left & +ve to right is sagging in nature.

$M_{AB} = 0$ (or) $M_A = 0$

$M_{BA} = M_{BC} = M_B = -11.18$ kNm.

$M_{CB} = 0$ (or) $M_C = 0$.

5) BMD

Max. free BM @ Span AB @ D $= \frac{10 \times 2 \times 3}{5}$

$= \frac{60}{5}$

$= 12$ kNm

Max free BM in Span BC @ E $= \frac{12 \times 6}{4}$

$= \frac{72}{4}$

$= 18$ kNm

Free BM diagram is shown in figure.

Now superpose the support moment diagram over the free BM diagram to get final BM diagram.

S.F diagram

1) Take moments about B to find the support reaction V_A and V_C

Consider left of

$B + 11.18 + (V_A \times 5) - (10 \times 3) = 0$ (or)

$V_A = 3.76\,kN \uparrow$

Consider right of B

$-11.18 - (V_C \times 6) + (12 \times 3) = 0$ (or)

$V_C = 4.14\,kN \uparrow$

(ii) From $\Sigma V = 0$ $V_B = (10+12) - 3.76 - 4.14$

$= 14.10\,kN \uparrow$

Now plot the SFD by Drawing force ordinates starting from the left. as shown in figure.

Alternative method. for finding the reaction

1) Reaction due to external load.

Span AB	Span BC
At A $= \frac{10 \times 3}{5} = 6\,kN \uparrow$	At B $= \frac{12 \times 3}{6} = 6\,kN \uparrow$
	$\frac{12 \times 3}{} = 6\,kN \uparrow$

A 60mm dia Solid ~~sun~~ circular Column is 3m long it is fixed at one end and free end other end. find the strength of the solid column? is the solid column is to be replaced by a column of the same area with a wall thickness of 10mm, Compute the strength of the hollow Column and find the ratio of the strength. $E = 2\times10^5\ N/mm^2$

Given data: circular column. wall $(t) = 10mm$

dia $(d) = 60mm$

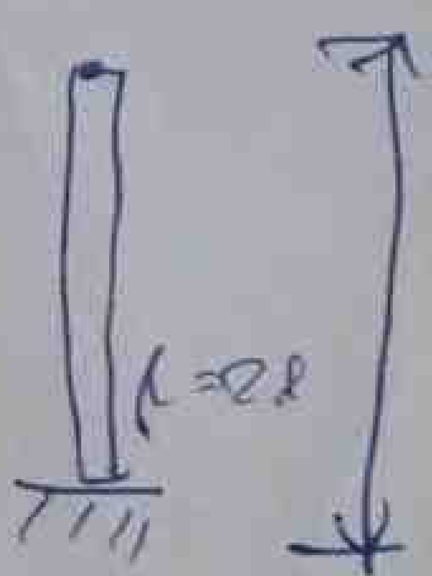
length $(l) = 3m = 3000mm$. $2\times3000 = 6000\ mm$

$E = 2\times10^5\ N/mm^2$.

To Find:

Strength of the solid column, ? P_E

Strength of the hollow column, $P_h = ?$



Soln:

ratio of Strength $= \frac{P_h}{P_s}$

$\cancel{P_E} = \frac{\pi^2}{l^2}$.

Euler's formula.

Strength of the solid column $P_s = \frac{\pi^2 EI}{l^2}$

$I = \frac{\pi d^4}{64} = \frac{\pi(60)^4}{64}$

$= \underline{63.61\times10^4\ mm^4}$

$P_s = \frac{\pi^2\times2\times10^5\times63.61}{6000^2}$

$= \cancel{139.5}\ \underline{34.87\times10^3 N}$.

(2) strength of the hollow column, P_h

$$P_h = \frac{\pi^2 E I}{l^2}$$

$$I = \frac{(D^2 - d^2)}{64}.$$

External dia, $D = d + 2t$

$= (d + 2 \times 10)$ $D(d+20)$

Area of the solid circular column $\Big\}$ = Area of the hollow circular column.

$$= \frac{\pi d^2}{4} = \frac{\pi (D^2 - d^2)}{4}$$

$$= \frac{\cancel{\pi}}{\cancel{4}} d^2 = \frac{\cancel{\pi}}{\cancel{4}} \cdot (D^2 - d^2)$$

$$d^2 = \left[(d + 20)^2 - d^2\right].$$

$$d^2 = (d + 20 + d)(\cancel{d} + 20 - \cancel{d})$$

$$60^2 = (2d + 20)(20)$$

$$60^2 = (40d) + (20) 400$$

$$3600 = 40d + 400$$

$$3600 - 400 = 40d \quad \therefore \frac{3200}{40} = 80$$

$$3200 = 40d \qquad d = 80\text{MM}.$$